The Impact of Government Inspections on Farms' Adulteration Behaviors in Co-Existing Traceable and Non-Traceable Supply Chains

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Problem definition: Economically motivated adulteration (EMA) frequently occurs in upstream segments of farming supply chains, posing significant challenges for downstream government inspections and underscoring the importance of supply chain traceability. ***Methodology/results***: In this paper, we develop a model to examine the impact of government inspections and penalty imposition on deterring EMA in settings with co-existence of traceable and non-traceable supply chains. In the traceable supply chain, the provenance of sampled products can be preciously targeted, allowing government penalties to be imposed on the adulterating upstream farm; While in the non-traceable supply chain, penalties can only be imposed on the downstream vendor. We first fully characterize the equilibrium adulteration behavior of farms in each supply chain, and analyze how government penalties, quality enhancement after adulteration, and other market parameters jointly impact the adulteration equilibrium. Our analysis suggests that higher government penalties may inadvertently induce the traceable farm to engage in adulteration. This behavior is driven by the traceable farm's desire to achieve a more favorable competitive position and circumvent the negative side-effect of indirect penalties from the non-traceable supply chain. We also conduct a preliminary empirical analysis by utilizing a sampling test dataset of China's domestic agricultural market. Under specific market conditions, we observe a positive correlation between government inspection frequency and adulteration rate of traceable farms, which aligns with our analytical results. ***Managerial implications***: Our results highlight the limitations of random inspection policies across traceable and non-traceable supply chains in a competitive market. By focusing inspection efforts on traceable products with specific characteristics, agencies can allocate resources more effectively and address EMA risks in farming supply chains more proactively.

Key words: economically motivated adulteration; government inspection; competing farming supply chains; supply chain traceability

1. Introduction

Food safety and adulteration risks are widespread issues and essential to everyone's well-being. The illegal use of adulterants, whether unintentionally or intentionally, at various stages of food manufacturing, processing, or distribution, constitutes a significant menace to public health. This paper focuses on the intentional adulteration that is pursued for a greater economic income, i.e., economically motivated adulteration (EMA, Levi et al. 2020b). One of the major drivers of EMA is output quality uncertainty, as addition of adulterant can increase the apparent quality of outputs or reduces the production cost (Johnson 2014). For instance, carcinogenic substances like Sudan Dyes have been employed to enhance the visual appeal of chili powders and curries, with the aim of attracting a larger customer base (Tarantelli 2017). Moreover, inherent characteristics of farming supply chains, including complex structures, opaque sourcing, and numerous intermediaries, further exacerbate EMA risks, which have created a multitude of incidents and economic losses. For example, in 2018, China reported that excessive and illicit utilization of food adulterants, including preservatives, sweeteners, and colorants, accounted for 23.85% of all substandard samples (China Quality News 2019). Globally, EMA affects 1% of the entire industry, resulting in annual losses ranging from $10 billion to $15 billion[1].

Consumers, often lacking expertise in product ingredients, struggle to discern adulterated items and remain unaware of EMA risks unless serious adulteration incidents break out (Cruse 2019). Hence, the primary responsibility for safeguarding food safety and public health against adulteration lies with government agencies. Typically, governments establish regulations governing the permissible usage levels of specific adulterants (e.g., preservatives, antibiotics) with defined dosage limits, and stipulate penalties for non-compliance. In order to enforce adherence to these regulations, government agencies conduct inspections by randomly taking sample products from the market, and subjecting them to testing for compliance with the specified standards. Failure to pass these tests leads to fines or (temporary) business bans on firms (Clever 2015).

Adulteration primarily takes place at the upstream stages of farming supply chains, such as farms and manufacturers, as highlighted by Jin et al. (2021a) and Huang et al. (2018). However, inspections are typically conducted downstream, in areas like wholesale markets, wet markets, and retail/supermarkets. In practice, it is often impractical for governments to regularly collect samples directly from the sources of adulteration. Additionally, imposing penalties when adulteration is detected is challenging due to the opaque nature of supply sources, particularly in supply chains with dispersed origins in remote rural areas of developing countries. This issue is further compounded by the fact that various government agencies responsible for different segments of farming

[1] https://www.fda.gov/food/compliance-enforcement-food/economically-motivated-adulteration-food-fraud

supply chains are highly decentralized, with misaligned interests and minimal coordination (Dong et al. 2022). For example, the Ministry of Agriculture and Rural Affairs of China (CMOA) oversees production activities on farms, such as seeding and plant cultivation, but places minimal emphasis on food safety concerns[2]. While the Administration for Market Regulation of China (CAMR) is tasked with ensuring food safety and conducting sampling tests on products, focusing mainly on the circulation stage. Finally, regulatory resources for inspections, such as human resources and annual budgets, are often limited, even in developed nations. For instance, only 1%-2% of the U.S. imported food shipments can be sampled annually (Racino 2011). The scarcity of inspection resources further weakens the effectiveness of government inspections in deterring adulterations.

To address the challenges posed by separated adulteration sources and sampling locations, decentralized government branches, and limited regulatory resources, the government is incentivized to tackle food safety issues by implementing supply chain traceability. For example, the Chinese government encourages farms, usually those with higher output quality, to enroll in a source origin certification system that records the provenance information of the products[3], and offer the information to appeal more consumers. The implementation of supply chain traceability and source origin certification systems could empower government agencies to maintain ongoing inspections in the downstream parts of supply chains, such as wholesale markets and wet markets. These markets serve as significant consolidation points and account for 70%-80% of China's total agricultural market (Jin et al. 2021a). If the sampling test fails and adulteration is detected, the government can target the source of adulteration through supply chain traceability, and penalties can be imposed on the corresponding farm. Under such a mechanism, the government agency can efficiently allocate its limited resources, as there is no need to perform sampling tests in the upstream parts, and strong connections or coordination with other government branches are not imperative. As a natural expectation, higher government penalties, such as more frequent inspections or increased amount of penalties, could deter adulteration in traceable supply chains.

In practical agricultural business settings, implementing a traceability system is costly, leading to incomplete coverage across supply chains. In the absence of traceability within supply chains, penalties for adulteration are exclusively imposed on the sampled vendor in wholesale/wet markets due to lack of officially documented sources of adulteration (Jin et al. 2021a)[4]. Consequently,

[2] http://english.moa.gov.cn/overview/202006/t20200601_300455.html

[3] http://www.moa.gov.cn/nybgb/2021/202108/202111/t20211104_6381383.htm

[4] This penalty mechanism is also supported by the Food safety law in China (Clever 2015). In more detail, Article 136 states that "In the event that the food producer or distributor who has performed its obligations in respect of inspecting purchased food under this Law, is not aware that the purchased food is inconsistent with food safety standards as supported by sufficient evidence, and is able to truthfully indicate the source, such food producer may be exempted from punishment,..."

Table 1: Failure Rate of Products in Traceable and Non-Traceable Supply Chains

	No. of sampling records	Pass the sampling test or not		Failure rate
		Yes	No	
Traceable	28216	24425	3791	13.4%
Non-traceable	15920	14909	1011	6.4%
Total	44136	39334	4802	10.9%

Notes. Details of the dataset are introduced in §5 and Gao et al. (2022).

the disincentive for non-traceable upstream farms to engage in adulteration due to higher government penalties is not that apparent, necessitating a closer examination of the penalty's impact on the entire supply chain. Additionally, the co-existence of traceable and non-traceable supply chains in the market adds dynamic complexity and introduces competition, further influencing the strategic adulteration behaviors of farms. Table 1 illustrates a preliminary statistical analysis of an adulteration test dataset on China's prefectural-level wholesale/wet markets. Unexpectedly, the adulteration rate in this dataset of products sold in traceable supply chains is more than twice higher than that in non-traceable ones. With these strategic and data-analytic considerations, this paper first identifies key drivers of traceable and non-traceable farms' adulteration behaviors. We then investigate the effectiveness of government inspections in the downstream parts of supply chains equipped with traceability to deter adulteration in the upstream parts, and study the impact of different market parameters on it.

We develop a game-theoretical model, which captures the central characteristics of farming adulteration and government inspection systems, to study the effectiveness of government inspections and penalties in deterring the adulteration behavior of farms in competing settings. We consider a game between two supply chains and a government agency. One of the supply chains is traceable, while the other is non-traceable, each consisting of one farm and one vendor. The initial average output quality of the traceable supply chain is higher, albeit incurring extra production costs. Farms within each supply chain are motivated to engage in adulteration to enhance the quality of their outputs, subsequently selling these substitute products to consumers through their respective vendors. The consumers are both quality- and price-sensitive, yet remain unaware of any adulteration. The government agency, responsible for conducting sampling tests on products sold by vendors, will impose penalties if adulteration is detected. Specifically, for the traceable supply chain, the adulterating farm can be accurately targeted and directly penalized. In contrast, for the non-traceable supply chain, the penalty can only be levied on the vendor. The non-traceable vendor also has an outside sourcing option if the expected government penalty is so high that it's not profitable to do business with the non-traceable farm.

Our model first reveals that the mechanisms through which government penalties affect adulteration in traceable and non-traceable supply chains are distinct. The impact of the *direct penalty* imposed on the adulterating traceable farm is straightforward. A higher direct government penalty could prompt the traceable farm to transition from adulteration to unadulteration, as it diminishes the competitive benefit of higher quality achieved through adulteration. In contrast, for the adulterating non-traceable farm, the penalty is applied to the downstream vendor. It does not influence the strategic decision of the non-traceable farm until the penalty reaches a level, at which the vendor would contemplate sourcing from the outside option. In order to retain the non-traceable vendor in the market, the non-traceable farm deviates from the revenue-maximizing wholesale price and concedes a lower wholesale price to the non-traceable vendor, which results in an *indirect penalty* on the non-traceable farm. This indirect penalty is facilitated through supply chain contracting, and it also carries a *side-effect* to the traceable supply chain: even producing unadulterated products, the traceable farm is compelled to reduce wholesale price and endure a loss in profit due to competition between the supply chains.

We then fully characterize the equilibrium adulteration behaviors of farms in both types of supply chains in response to government penalties. Specifically, we investigate how the equilibrium behavior of the traceable farm shifts as government penalties increase. Generally, higher government penalties deter farms from adulterating. However, there are also situations in which higher government penalties lead the traceable farm to resort to adulteration. It is a result of an equilibrium where the traceable farm adulterates and the non-traceable one does not. This equilibrium emerges in three possible parameter regions for two different reasons. First, when government penalties are not high and the indirect penalty on the non-traceable farm has not yet taken effect, the traceable farm prefers to adulterate, accepting the direct penalty to gain a stronger competitive position. Second, when government penalties are relatively high and the indirect penalty on the non-traceable farm becomes effective, the negative side-effect on the traceable farm, resulting from deviations in the revenue-maximizing wholesale price, escalates significantly with increasing government penalties. Consequently, the traceable farm is compelled to engage in adulteration to avoid the severe side-effect, which could even result in being pushed out of the market. We also explore how various market parameters influence the three regions in which the traceable farm engages in adulteration while the non-traceable farm does not. Naturally, the regions expand for higher quality enhancement after adulteration, and contract with greater initial quality differences in the outputs of these two supply chains (i.e., a stronger incentive for the non-traceable farm to adulterate). For the specific region where the traceable farm engages in adulteration due to the side-effect of the indirect penalty, this phenomenon only occurs when production costs are low, which results in a significant positive correlation between higher government penalty and traceable farm's adulteration behavior. This counter-intuitive result is supported by a preliminary empirical analysis, which is conducted based on a sampling test dataset of China's domestic agricultural product market.

The remainder of this paper is organized as follows. In §2 we review the related literature, and in §3 we introduce the model formulation. We then derive the adulteration equilibrium for the traceable and non-traceable farms in §4, and provide preliminary empirical evidence on the impact of higher government penalty on the adulteration rate of traceable farms in §5. In §6 we highlight our insights and conclude the paper. All proofs and technical lemmas are provided in the Online Appendix, which also includes a table of notations.

2. Literature Review

Our paper is related to three streams of literature in operations and supply chain management. The first research stream studies socially responsible operations, which focus on various methods (e.g., contracts, monitoring, or inspection policies) adopted by manufacturers to motivate suppliers' socially responsible behavior. For example, Guo et al. (2016) analyze the impact of four sourcing strategies to improve supply chain responsibility, where suppliers can be responsible and risky, and consumers' social consciousness is heterogeneous. Chen and Lee (2017) show the effectiveness of supplier certification, process audits, and contingency payment on better supplier compliance to social and environmental standards. Cho et al. (2019) investigate firms' inspection and wholesale pricing strategies to combat child labor in global supply chains. Orsdemir et al. (2019) examine how vertical integration of supply chain partners can eliminate firms' social and environmental responsibility risks. Kraft et al. (2020) study a firm's investment and information disclosure strategy to motivate the supplier's social responsibility practices when the firm has limited visibility. The socially irresponsible behaviors in these models do not impact the functionality quality of the outputs, while the EMA adopted by farms endogenously influences the product quality.

Within the field of socially responsible operations, a handful of papers focus on settings in developing economies and study issues of safety and adulteration risks in agricultural supply chains; see Chen et al. (2021) for a comprehensive review. Babich and Tang (2012) study three mechanisms, deferred payment, inspection, and a combined mechanism, to deal with suppliers' adulteration problems. They show deferred payment can completely deter adulteration, while inspection cannot. Levi et al. (2020b) develop a new framework to investigate preemptive and reactive EMA risks in farming supply chains and analyze how quality uncertainty, supply chain dispersion, and test sensitivity jointly impact farms' adulteration behavior. Their results also highlight the limitations of end-product inspection by the manufacturer. Mu et al. (2014, 2016, 2019) focus on milk supply chains in developing countries and design testing policies and incentive schemes to address the adulteration risk, with driving forces like high testing cost, station competition, and farmers free-riding. Similar to the above models, we also study the effectiveness of a specific mechanism or strategy in deterring adulteration. However, our model moves the perspective of downstream manufacturers or buyers to the one of governments, as related to the following stream of literature.

Second, governments or nongovernmental organizations (NGOs) also play an increasingly important role in agricultural supply chains to achieve goals such as improving farm welfare and reducing income disparities, through different policy instruments like subsidy or taxes (Akkaya et al. 2021, Guo et al. 2022, Levi et al. 2022, Fan et al. 2023), market information provision (Tang et al. 2015, Zhou et al. 2021), and guaranteed support prices (Guda et al. 2021). The papers more related to our work study the government/NGO's intervention in farming supply chains to maintain supply chain sustainability and output safety, and research in this field typically examines two topics. The first topic is the strategic role of government/NGO in ensuring food safety. For example, Kraft et al. (2013) study whether an NGO should target the industry or the regulatory body to remove a potential hazard from being used. Dong et al. (2022) compare the food safety risks and system payoffs of centralized and decentralized auditing structures, and show that the change from decentralization to centralization may not improve food safety.

Another topic on government/NGO intervention studies how to utilize public sample data to analyze supply chain adulteration risks and develop better policies to improve inspection efficiency. For example, Huang et al. (2018) leverage datasets to quantify supply chain dispersion and regional government strength, and study their joint influence on EMA risks in China's farming supply chains. Levi et al. (2020a) leverage historical imported food data of the U.S. and develop a data-driven approach to build a predictive risk model, which provides recommendations on high-risk firms to test in the future. Their results suggest that supply chain feature based risk analytics could significantly improve the effectiveness of site inspection. Jin et al. (2021a) leverage a large self-constructed dataset to identify the source of adulteration risks in China's food supply chain. Their result highlights potential gaps in the current test allocation policy and suggests reallocating scarce regulatory resources to the high-risk parts of supply chains. Such data-driven studies have limitations to implement (e.g., data integration and operation) in actual practice, and our study complements this stream of literature by showing the effectiveness of regular government inspection with supply chain traceability.

Finally, our paper contributes to the research stream on the operational impacts of supply chain traceability. Previous studies in this field regard traceability as an instrument to help firms build reputations (Saak 2016), allocate liability (Piramuthu et al. 2013), and combat label misconduct (Yao and Zhu 2020), and examine factors that influence the adoption of traceability (Jin et al. 2021b). While these papers mainly show the positive effect of supply chain traceability, some papers investigate its negative effect. For example, Resende-Filho and Hurley (2012) show that contingent payment can be a substitute for traceability precision and vice-versa, so higher precision of either voluntary or mandatory traceability system may not be a credible signal of safer food. In the multi-tier supply chain model of Dong et al. (2023), buyers exploit the upstream suppliers more, and suppliers are less incentivized to reduce contamination risk after adopting blockchain-enabled traceability. Therefore, improved traceability might hurt food safety and agents' profits. Cui et al. (2023) investigate the quality contract equilibrium between one buyer and two suppliers under different supply chain structures. They show that the flexible product call enabled by traceability-driven blockchain could reduce product quality and hurt suppliers' profits in parallel supply chains. Our model also reveals the potential drawback of traceability in a competing farming business setting, where more regulatory inspection might stimulate adulteration in traceable supply chains.

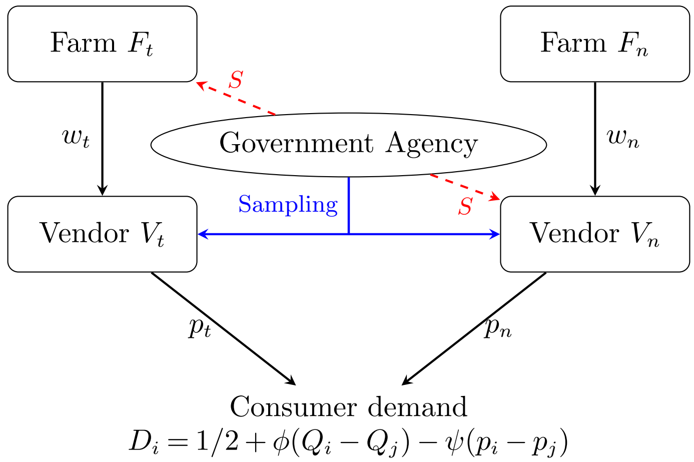
3. Model

In this section, we develop a game-theoretic model to study the role of supply chain traceability and government inspections in deterring adulteration in agricultural business settings. The model involves two competing supply chains, each comprising a single farm (he) and a downstream vendor (she). Each farm exclusively supplies his corresponding vendor. The weakly self-disciplined farms, who face outputs quality uncertainty, could engage in adulteration to decrease the probability of low-quality output, i.e., the preemptive adulteration described in Levi et al. (2020b). The vendors, responsible for supplying end consumers, operate within the same wet market or wholesale market. Within this market, a government agency conducts random sampling tests on the products sold, and we focus on the perfect testing scenarios[5] in this paper. If adulteration is detected, the government agency has the power to fine the player, contingent on the traceability of the supply chains. Here traceability/non-traceability in our setting means whether the vendor can provide an official document indicating the adulterated agricultural products are sourced exactly from the corresponding farm. Given that not all supply chains in real agricultural business have traceability, we assume that one of the supply chains in our model is traceable (F_t and V_t denote the farm and vendor, respectively), and the other one is non-traceable (F_n and V_n denote the farm and vendor, respectively).

Figure 1 shows the main structure of our model. Note that the "farm" notation in our model represents more generally an upstream player in the supply chains, and similarly, the "vendor" notation represents a player in the downstream of the supply chains (e.g., a wholesaler or a retailer in the circulation stage). Although agricultural business networks in practice could be more complex, including more agents and connections between them, this parsimonious model captures the primary role of supply chain traceability and government inspections in deterring adulteration, as we will discuss in detail next.

[5] Under perfect testing scenarios, the adulterant of a certain product is well known, and accurate methods exist to test whether the food safety standards are obeyed. See Levi et al. (2020b) for more discussions on perfect/imperfect testing scenarios.

Figure 1: Model with Two Competing Farming Supply Chains and a Government Agency



3.1. Farms

Following Levi et al. (2020b), we assume the quality of a farm's output is uncertain ex ante. The probability of each unit is high quality (q_H) with probability θ_i^H and low quality (q_L, $q_H > q_L > 0$) with probability $1 - \theta_i^H$, $0 < \theta_i^H < 1$ and $i \in \{t, n\}$. Consequently, the unit product quality is a random variable that follows a Bernoulli distribution, and the expected quality is $Q_{i0} = q_H \theta_i^H + q_L(1 - \theta_i^H)$. We assume $\theta_t^H > \theta_n^H$, i.e., the traceable farm has a higher initial probability of producing high-quality units, and he will incur an extra unit production cost $c > 0$. Without losing generality, we also assume that the production cost of the non-traceable farm is (normalized to) 0. The assumptions are based on facts that traceable farms are willing to make more efforts in production, such as following more strict production criteria and processing better agricultural inputs, which incurs extra cost and results in a higher probability of high-quality units (Xu et al. 2020).

In our model, output quality is defined as product attributes, such as appearance, texture, and flavor, that are observable and can impact the consumers' evaluation of purchasing utility. In order to increase the observable performances of the outputs, both farms can choose to adulterate before the quality realization of their outputs. Let $x_i \in \{A, U\}$, $i \in \{t, n\}$, denote the adulteration decisions of traceable and non-traceable farms, with $x_i = A$ representing adulteration and $x_i = U$ representing unadulteration. Adding the adulterants can increase the likelihood of producing high-quality outputs. Under the perfect testing of the government inspection, the farm will adulterate with the maximum dosage if he decides to engage in preemptive adulteration, and the probability of producing fake high-quality products will increase to a maximum level (Levi et al. 2020b). Therefore, we assume the probability that each unit of output is high quality, when (traceable or non-traceable) farm adulterates, will increase to a maximum level, θ^{max}, and $0 < \theta_n^H < \theta_t^H < \theta^{max} < 1$. As a result, the expected quality for each adulterated unit is $Q_A = q_H \theta^{max} + q_L(1 - \theta^{max})$, and $Q_A > Q_{t0} > Q_{n0}$. This expected quality difference is an important motivation for farms to engage in adulteration, especially for the non-traceable one, as higher product quality results in higher revenue in the retail market.

After the production stage, each farm simultaneously announces the wholesale price w_i for the downstream vendor. We assume the vendors are wholesale price-takers because they are small-sized entities in wholesale/wet markets and hold a relatively weak position in supply chains. On the other hand, farms make decisions on the wholesale prices rather than the production quantity because the outputs of the two supply chains are non-commodity, and their qualities are differentiated. In the Online Supplement D, we release these assumptions and study a model where wholesale prices are proportional to output quality with an exogenously given ratio, and our main results still hold.

3.2. Vendors

The vendors in each supply chain can audit the production process of the upstream farm, thereby gaining insights into their decisions regarding adulteration, and receiving the wholesale price from the corresponding farm. Given the adulteration decisions and wholesale prices of the upstream farms, the vendors first decide whether or not to source from the upstream farm. If they choose to do so, they then establish the retail price of the agricultural products, denoted as p_i, $i \in \{t, n\}$, respectively; Conversely, if they opt not to procure from the upstream farm, vendors can procure from an outside option, whose products are guaranteed to be safe (Dong et al. 2022). We normalize the payoffs of vendors and upstream farms to 0 if there is no trade between them.

To simplify the analysis without losing key insights, we normalize the auditing and processing costs of the vendors to 0 so that the wholesale prices are the only costs for each vendor in the supply chains. Additionally, farming and manufacturing stages are the major sources of adulteration (Jin et al. 2021a), so we assume the vendors in our model do not engage in adulteration. However, they are also highly economically motivated and care little about consumers' health, i.e., hold a weak sense of social responsibility. They will choose to procure from farms and sell the products as long as they are profitable, regardless of whether the products are adulterated or not. Similar settings have been studied in agricultural business where the retailer sells adulterated products (e.g., Mu et al. 2016 and Dong et al. 2022).

Consumers in our model can not discern whether the products contain adulterants, and they barely have the option to do a sampling test. As a result, their purchasing decisions are primarily influenced by the average quality of the outputs offered by vendors. The realized number of high-quality units for a particular batch of each farm's outputs follows a Binominal distribution, and the output average quality is just the expected quality of each unit, i.e., Q_{i0} or Q_A. Following Balasubramanian and Bhardwaj (2004) and Matsubayashi and Yamada (2008), we assume consumer demand for vendor V_i is positively (negatively) related to the average quality of her (competing vendor V_j's) outputs and negatively (positively) related to her (competing vendor V_j's) retail price, which is specified as follows:

$$D_i(x_t, x_n) = 1/2 + \phi\left(Q_i(x_i) - Q_j(x_j)\right) - \psi(p_i - p_j), i, j \in \{t, n\}, i \neq j, \tag{1}$$

where $Q_i(U) = Q_{i0}$ if the farm did not adulterate, and $Q_i(A) = Q_A$ if adulterated. ϕ and ψ are quality- and price-sensitive parameters of customers. To exclude some trivial scenarios, we assume the market parameters satisfy certain constraints so that consumer demand for each vendor is positive through our following analysis. We also assume the profit surplus because of higher initial quality can cover the extra production cost of traceable products, so the farm is willing to implement the traceability system, i.e.,

$$\phi(Q_{t0} - Q_{n0}) > \psi c. \tag{2}$$

3.3. Government Agency

To ensure adherence to regulations and standards, and maintain food safety, the downstream government agency, which has little coordination with upstream branches, regularly takes samples from the circulation stage of the supply chains, such as wholesale/wet markets, and conducts tests to detect any illegal adulterants. Once the sampling test fails, i.e., adulteration is caught, the government agency can fine the vendor or the upstream farm (source of the problematic products, if he can be traced), and the expected government penalty S is a product of inspection probability and the amount of penalty charged per time adulteration caught (Mu et al. 2016). We assume both of them are exogenously given. First, the penalty amount for each instance of adulteration is set by the country's legislative body and is mandated to be uniform nationwide. Second, the probability of inspection for a certain vendor depends on the inspection frequency of government agencies and the probability of the vendor being sampled per inspection. The inspection frequency is mainly restricted by the scarce regulatory resources of the (local) government agency. The probability of the vendor being sampled per inspection is the same between the two supply chains because the agency randomly takes samples from the two vendors. Therefore, we assume a uniform government penalty S over the two supply chains, and we will discuss this assumption further in Online Appendix §A.2.

While inspections take place in the circulation stage of the supply chains, it is the production stage that serves as the primary source of adulteration. This disparity prompts the government to formulate a penalty mechanism that differs between supply chains. Specifically, in the traceable supply chain, the vendor can provide officially certified documents and labels verifying the origin of the problematic products. Consequently, the government penalty can be directed towards the corresponding farm, which cannot refute it. In contrast, within the non-traceable supply chain, the vendor is unable to convincingly determine the source of the adulterated products. As a result, she must bear the government penalty herself. Our model effectively incorporates the critical role of traceability in helping the government to assign liability accurately by identifying the party responsible for the adulteration behavior.

3.4. Sequence of Events

All players in our model (except the government agency) are risk-neutral and aim to maximize their expected profits. We do not consider the players' strategic decision on implementing the traceability system in this paper, but take it as existing in the traceable supply chain and the cost of implementation is sunk. Table A.1 summarizes all the notations used in this model and the sequence of events is as follows.

Stage I: Each farm simultaneously and individually decides whether to adulterate, and then the average quality of each farm's output is realized. *Stage II:* Farms then announce the wholesale price for the downstream vendors, respectively. Each vendor audits the corresponding farm's adulteration behavior, receives the wholesale price, and decides whether to procure products from the upstream farm or source from the outside option. Each vendor then sets up the retail prices for consumers after the sourcing decisions are made. *Stage III:* Consumers buy products given the average quality and retail prices of outputs in each supply chain. *Stage IV:* The government agency conducts sampling tests on the products sold in the market. If adulteration is caught, a penalty will be imposed on the traceable farm or the non-traceable vendor.

The described model settings above reflect existing adulteration and inspection practices in some developing economies (e.g., China). We also provide some justifications for the assumptions of our model (information structure, market reaction, and uniform government penalty) in Online Appendix §A.2 due to space limitation.

4. Equilibrium Analysis

Based on the model in the previous section, the expected profit functions of the vendors are as follows:

$$\Pi_{V_t}^{x_t x_n} = \pi_{V_t}^{x_t x_n}, \tag{3}$$
$$\Pi_{V_n}^{x_t x_n} = \max\{\pi_{V_n}^{x_t x_n} - S \cdot \mathbb{1}_{\{x_n = A\}},\ 0\}, \tag{4}$$

where the superscript $x_t x_n$ specifies the respective adulteration decisions of the traceable and non-traceable farms, $x_t, x_n \in \{A, U\}$. $\pi_{V_i}^{x_t x_n} = (p_i - w_i) D_i(x_t, x_n)$, $i \in \{t, n\}$, are the revenue functions of the vendors without considering the government penalty, and $D_i(x_t, x_n)$ is characterized in Equation (1). The indicator function $\mathbb{1}_{\{x_n = A\}} = 1$ when $x_n = A$, i.e., the non-traceable farm chooses to adulterate; otherwise, it's 0. For the traceable vendor V_t, she does not need to source from the outside option as the government penalty will not be imposed on her. In contrast, for the non-traceable vendor V_n, she will not source from the adulterating upstream farm if the government penalty surpasses her revenue. Instead, she will source from the outside option, under which her profit is normalized to 0. Similarly, the expected profit functions of the farms, with adulteration decisions (x_t, x_n), are as follows:

$$\Pi_{F_t}^{x_t x_n} = \pi_{F_t}^{x_t x_n} - S \cdot \mathbb{1}_{\{x_t = A\}}, \tag{5}$$

$$\Pi_{F_n}^{x_t x_n} = \begin{cases} \pi_{F_n}^{x_t x_n} & \text{if } V_n \text{ procures from } F_n, \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

where $\pi_{F_t}^{x_t x_n} = (w_t - c) D_t(x_t, x_n)$ is the revenue of the traceable farm without considering government penalty, and $\pi_{F_n}^{x_t x_n} = w_n D_n(x_t, x_n)$ is the revenue function of the non-traceable farm if the non-traceable vendor procures from him. The indicator function $\mathbb{1}_{\{x_t = A\}} = 1$ when $x_t = A$, i.e., the traceable farm chooses to adulterate; otherwise, it's 0. In our model, the traceable farm's decision on whether or not to adulterate is driven by the trade-off between the expected payoff gain (competitive advantage because of higher output quality) and the penalty of adulteration from the government. While for the non-traceable farm, apart from the profit gain of improved quality, his adulteration decision is also affected by the procurement decision of the non-traceable vendor. The profit of the non-traceable farm would be 0, if the government penalty allocated to the vendor is so high that the non-traceable vendor decides not to procure from him. To simplify our notation in the following analysis, we use $\Delta q = Q_{t0} - Q_{n0}$ to denote the initial expected quality difference between outputs of the traceable and non-traceable supply chains and let $r = Q_A - Q_{t0}$ denote the traceable product's expected quality enhancement after adulteration. Naturally, $\Delta q + r$ is the non-traceable product's expected quality enhancement after adulteration.

4.1. Sub-game Perfect Equilibrium in Each Scenario

Following backward induction, we first find the subgame equilibria of Stage II-IV for all possible farms' adulteration strategies of Stage I, and then solve for the equilibrium of Stage I. There are four possible scenarios of the farms' adulteration strategies: neither farm adulterates (U, U), the traceable farm adulterates and the non-traceable one does not (A, U), the traceable farm does not adulterate and the non-traceable one adulterates (U, A), and both farms adulterate (A, A). In each scenario, we first find the optimal retail and wholesale prices and derive the sub-equilibrium profits for the players ($\Pi_{F_i}^{x_t x_n *}$ and $\Pi_{V_i}^{x_t x_n *}$, $i \in \{t, n\}$). Then we demonstrate how each farm's adulteration behavior in Stage I changes with respect to the government penalty and other market parameters, given his opponent's adulteration strategy.

4.1.1. Scenario (U,U): Neither Farm Adulterates.

In this scenario, the average quality of outputs for traceable and non-traceable supply chain is $Q_t(U) = Q_{t0}$ and $Q_n(U) = Q_{n0}$, respectively, so $Q_t(U) - Q_n(U) = \Delta q$, i.e., the traceable supply chain keeps the quality advantage of Δq. Demands for each vendor following Equation (1), are given as:

$$D_t(U, U) = \tfrac{1}{2} + \phi \Delta q - \psi(p_t - p_n), \quad D_n(U, U) = \tfrac{1}{2} - \phi \Delta q + \psi(p_t - p_n). \tag{7}$$

Given the adulteration decisions and wholesale prices of the farms, it is straightforward to show the concavity of each vendor's revenue $\pi_{V_i}^{UU}$ on p_i, $i \in \{t,n\}$, and the optimal retail prices are

$$p_t^* = \frac{3+2\phi\Delta q+\psi(4w_t+2w_n)}{6\psi}, \qquad p_n^* = \frac{3-2\phi\Delta q+\psi(2w_t+4w_n)}{6\psi}. \tag{8}$$

Anticipating the vendors' pricing strategies, farms set their optimal wholesale price as follows:

$$w_t^* = \frac{9+2\phi\Delta q+4\psi c}{6\psi}, \qquad w_n^* = \frac{9-2\phi\Delta q+2\psi c}{6\psi}. \tag{9}$$

Each player's optimal revenue can be obtained by plugging the optimal pricing decisions ((8) and (9)) into the corresponding revenue functions. Since neither farm adulterates, there is no government penalty for the players. Therefore, the sub-equilibrium profits of the players in each supply chain are the same as their optimal revenues, which are in the following forms,

$$\Pi_{F_n}^{UU*} = \pi_{F_n}^{UU*} = \frac{(9-2\phi\Delta q+2c\psi)^2}{108\psi}, \quad \Pi_{V_n}^{UU*} = \pi_{V_n}^{UU*} = \frac{(9-2\phi\Delta q+2c\psi)^2}{324\psi}. \tag{10}$$

$$\Pi_{F_t}^{UU*} = \pi_{F_t}^{UU*} = \frac{(9+2\phi\Delta q-2c\psi)^2}{108\psi}, \quad \Pi_{V_t}^{UU*} = \pi_{V_t}^{UU*} = \frac{(9+2\phi\Delta q-2c\psi)^2}{324\psi}. \tag{11}$$

According to the profit functions in (10) and (11), farms' profits are positively correlated with the quality advantage of their products. Hence, they have the incentive to seek higher quality by adulteration. On the other hand, a high production cost reduces the sub-equilibrium profits of players in the traceable supply chain and causes a competitive disadvantage for them. However, the assumption in Equation (2) guarantees that the players in the traceable supply chain are still more profitable than the ones in the non-traceable supply chain, which is worthwhile to implement traceability. Note that the profit of each farm is greater than that of the downstream vendor, specifically with a ratio of $3:1$, because the wholesale price in each supply chain is set by the farm, who is the Stackelberg leader in our setting.

4.1.2. Scenario (A,U): Traceable Farm Adulterates and Non-traceable Farm Unadulterates. In this scenario, the average quality of each farm's outputs is $Q_t(A) = Q_A$ and $Q_n(U) = Q_{n0}$, so $Q_t(A) - Q_n(U) = \Delta q + r$, i.e., the traceable farm expands the quality advantage over the non-traceable farm by adulteration. Following the same procedures as the scenario above, we can derive the optimal revenue functions of the farm and vendor in each supply chain $\pi_{F_i}^{AU*}$ and $\pi_{V_i}^{AU*}$, $i \in \{t,n\}$, respectively. Given the non-traceable farm not adulterating, there will be no government penalty for the non-traceable vendor, and she always procures the agricultural products from the upstream farm. Their sub-equilibrium profits in this scenario are

$$\Pi_{F_n}^{AU*} = \pi_{F_n}^{AU*} = \frac{(9-2\phi(\Delta q+r)+2\psi c)^2}{108\psi}, \quad \Pi_{V_n}^{AU*} = \pi_{V_n}^{AU*} = \frac{(9-2\phi(\Delta q+r)+2\psi c)^2}{324\psi}. \tag{12}$$

Because of supply chain traceability, there will be a government penalty for the traceable farm, and the sub-equilibrium profits for players in the traceable supply chain are

$$\Pi_{F_t}^{AU*} = \pi_{F_t}^{AU*} - S = \frac{(9+2\phi(\Delta q+r)-2\psi c)^2}{108\psi} - S, \quad \Pi_{V_t}^{AU*} = \pi_{V_t}^{AU*} = \frac{(9+2\phi(\Delta q+r)-2\psi c)^2}{324\psi}. \tag{13}$$

Equation (13) shows the traceable farm's sub-equilibrium profit decreases in the government penalty, and it will overcome the profit premium brought by the quality advantage of adulteration. Let $S_{F_t}^U = \pi_{F_t}^{AU*} - \pi_{F_t}^{UU*} = \frac{\phi r(9+2\phi\Delta q+\phi r-2\psi c)}{27\psi}$ be the difference between the traceable farm's optimal revenue under scenarios (A,U) and (U,U), and we have the following lemma shows the impact of government penalty on his adulteration decision.

LEMMA 1. *When $S > S^U_{F_t}$, $\Pi^{AU*}_{F_t} < \Pi^{UU*}_{F_t}$, the traceable farm does not adulterate, in case of an unadulterating non-traceable farm; Otherwise, the traceable farm adulterates when $0 < S \leq S^U_{F_t}$.*

Lemma 1 indicates a *direct penalty mechanism* that takes into effect in deterring the traceable farm from adulteration. Despite the government sampling inspection and the sources of adulteration being situated at different parts of the supply chain (downstream and upstream, respectively), the government penalty can accurately target the farm involved in adulteration due to the traceability within the supply chain. Consequently, a sufficiently high government penalty can erode the illicit profit gain and effectively deter adulteration by the traceable farm.

4.1.3. Scenario (U,A): Traceable Farm Unadulterates and Non-traceable Farm Adulterates. In this scenario, the average output qualities of the farms are $Q_t(U) = Q_{t0}$ and $Q_n(A) = Q_A$, so $Q_n(A) - Q_t(U) = r$, i.e., the non-traceable farm now holds the competitive advantage on average output quality over the traceable farm. Unlike the traceable supply chain in which the farm gets the punishment, a government penalty will be allocated to the vendor in the non-traceable supply chain if the product is detected adulterated. Therefore, in Stage II, the non-traceable vendor compares the profits between sourcing from the corresponding farm ($\Pi^{UA}_{V_n} = \pi^{UA}_{V_n} - S$) and sourcing from the outside option (normalized to 0) and decides whether to stay or leave the market. The following lemma shows each farm's best-response wholesale price decision and characterizes the sub-equilibrium profits of the farms and vendors, respectively.

LEMMA 2. *In scenario (U,A), the sub-equilibrium wholesale prices (w^*_i) and profits ($\Pi^{UA*}_{F_i}$, $\Pi^{UA*}_{V_i}$) of the players change with respect to different government penalties.*

(i) If $0 < S \leq \tilde{\pi}^{UA}_{V_n}$, the sub-equilibrium wholesale prices and profits of players in each supply chain are

$$\tilde{w}_t = \frac{9-2\phi r+4\psi c}{6\psi}, \quad \tilde{w}_n = \frac{9+2\phi r+2\psi c}{6\psi}. \tag{14}$$

$$\tilde{\Pi}^{UA}_{F_n} = \tilde{\pi}^{UA}_{F_n} = \frac{(9+2\phi r+2\psi c)^2}{108\psi}, \quad \tilde{\Pi}^{UA}_{V_n} = \tilde{\pi}^{UA}_{V_n} - S = \frac{(9+2\phi r+2\psi c)^2}{324\psi} - S > 0. \tag{15}$$

$$\tilde{\Pi}^{UA}_{F_t} = \tilde{\pi}^{UA}_{F_t} = \frac{(9-2\phi r-2\psi c)^2}{108\psi}, \quad \tilde{\Pi}^{UA}_{V_t} = \tilde{\pi}^{UA}_{V_t} = \frac{(9-2\phi r-2\psi c)^2}{324\psi}. \tag{16}$$

(ii) If $\tilde{\pi}^{UA}_{V_n} < S \leq min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi})$, the sub-equilibrium wholesale prices and profits of players in each supply chain are

$$\bar{w}_t = \frac{3+\psi c-3\sqrt{S\psi}}{\psi}, \quad \bar{w}_n = \frac{9+2\phi r+2\psi c-12\sqrt{S\psi}}{2\psi}. \tag{17}$$

$$\bar{\Pi}^{UA}_{F_n} = \frac{\sqrt{S}(9+2\phi r+2\psi c)-12S\sqrt{\psi}}{2\sqrt{\psi}}, \quad \bar{\Pi}^{UA}_{V_n} = 0. \tag{18}$$

$$\bar{\Pi}^{UA}_{F_t} = \frac{3(\sqrt{S\psi}-1)^2}{\psi}, \quad \bar{\Pi}^{UA}_{V_t} = \frac{(\sqrt{S\psi}-1)^2}{\psi}. \tag{19}$$

(iii) If $S > min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi})$, there would be no transaction in the market, so $\hat{\Pi}^{UA}_{F_n} = 0$ and $\hat{\Pi}^{UA}_{V_n} = 0$; $\hat{\Pi}^{UA}_{F_t} = 0$ and $\hat{\Pi}^{UA}_{V_t} = 0$.

First, when the government penalty is low (Case (i) of Lemma 2, $0 < S \leq \tilde{\pi}_{V_n}^{UA}$), farms still choose the revenue-maximizing wholesale price, and the non-traceable vendor remains profitable in procuring from the non-traceable farm ($\tilde{\Pi}_{V_n}^{UA} > 0$), so she stays in the market. This case is similar to the scenarios (U,U) and (A,U), except the non-traceable vendor is burdened with the government penalty. In this case, the government penalty does not swim back to the upstream farm and has no strategic influence on the wholesale price decision of the non-traceable farm.

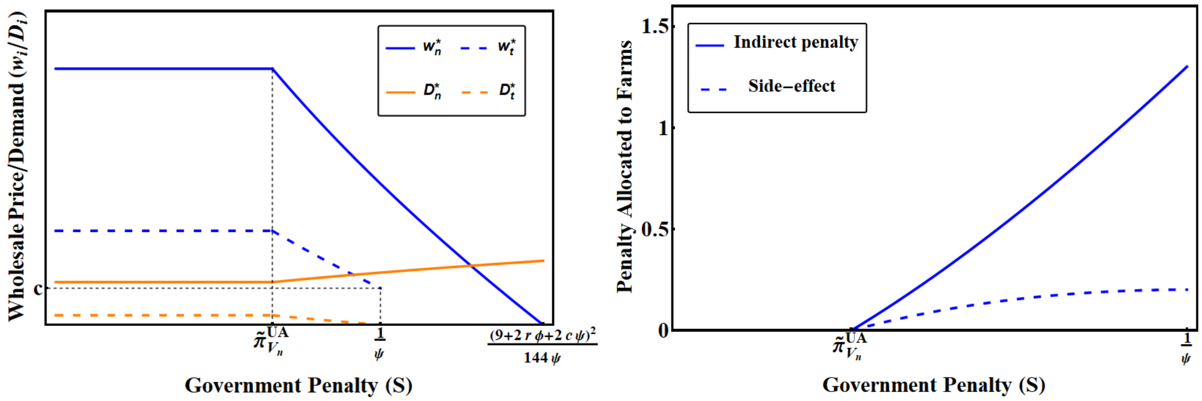
With the increase of government penalty, the sub-equilibrium profit of the non-traceable vendor would finally drop to 0. When $S > \tilde{\pi}_{V_n}^{UA}$, the non-traceable vendor will leave the market due to negative profit and source from the outside option if the non-traceable farm still sets the revenue-maximizing wholesale price. As a result, the profit of the non-traceable farm will also be 0. Hence, the non-traceable farm has to reduce the wholesale price w_n to keep the vendor in the market but remain her profit the same as the outside option, i.e., $\bar{\Pi}_{V_n}^{UA} = 0$. In this case, although the government penalty is not imposed on the non-traceable farm, it starts to affect his operational strategy because the non-traceable farm needs to deviate from the revenue-maximizing wholesale price and concedes a lower wholesale price to the downstream vendor, to keep her in the business. The higher the government penalty, the more the non-traceable farm must compensate the non-traceable vendor. Meanwhile, the traceable farm also reduces his wholesale price as a reaction to the stronger competition from the non-traceable one. Figure 2(a) shows the sub-equilibrium wholesale prices and demands of the farms in scenario (U,A) as the government penalty increases.

Finally, when the government penalty is very high (Case (iii) of Lemma 2, $S > min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi}))$, either the traceable farm is driven out of the market ($\bar{w}_t = c$ and $\bar{D}_t = 0$ when $S = \frac{1}{\psi}$), or the non-traceable farm prefers not to offer the wholesale price ($\bar{w}_n = 0$ when $S = \frac{(9+2\phi r+2\psi c)^2}{144\psi}$), so there is no transaction in the market. Consequently, the sub-equilibrium profits of all players are 0. The following lemma characterizes the threshold of government penalty on the non-traceable farm's adulteration strategy in Stage I, given the unadulteration of the traceable farm.

Lemma 3. *Given the traceable farm unadulterating, the non-traceable farm does not adulterate when* $S > min(S_{F_n}^U, \frac{1}{\psi})$, *i.e.,* $\Pi_{F_n}^{UA*} < \Pi_{F_n}^{UU*}$; *otherwise, the non-traceable farm adulterates when* $0 < S \leq min(S_{F_n}^U, \frac{1}{\psi})$, *where* $S_{F_n}^U$ *is the government penalty threshold satisfying* $\bar{\Pi}_{F_n}^{UA} = \Pi_{F_n}^{UU*}$, *and* $\tilde{\pi}_{V_n}^{UA} < S_{F_n}^U < \frac{(9+2\phi r+2\psi c)^2}{144\psi}$.

Lemma 3 indicates an *indirect penalty mechanism* that deters adulteration in the non-traceable supply chain. Although the government penalty can not be imposed on the non-traceable farm directly due to decentralized inspection and lack of supply chain traceability, it comes into effect through supply chain contracting. The non-traceable farm suffers from the distorted sub-equilibrium wholesale price when $S > \tilde{\pi}_{V_n}^{UA}$, in order to keep the downstream vendor sourcing from

Figure 2: The Effect of Government Penalty in Scenario (U,A) (Color Online)



(a) Effects on the wholesale prices and demands

(b) Indirect penalty and its side-effect

Notes. The parameters are $c=0.7$, $\Delta q=1$, $\psi=0.5$, $\phi=0.5$, $r=5$.

him. The indirect penalty, i.e., $\tilde{\Pi}^{UA}_{F_n}-\bar{\Pi}^{UA}_{F_n}$, which increases in the government penalty, could overcome the profit gain of adulteration so that the non-traceable farm prefers unadulteration. It also has a *side-effect* on the traceable farm. Although not adulterating, the traceable farm also suffers profit loss because of wholesale price reduction, which is $\tilde{\Pi}^{UA}_{F_t}-\bar{\Pi}^{UA}_{F_t}$. As shown in Figure 2(b), the indirect penalty for the non-traceable farm and its side-effect for the traceable farm emerge when $S>\tilde{\pi}^{UA}_{V_n}$, and increase in the government penalty.

4.1.4. Scenario (A,A): Both Farms Adulterate. In this scenario, the average quality of each farm's outputs is $Q_t(A)=Q_n(A)=Q_A$, so $Q_t(A)-Q_n(A)=0$, i.e., there is no average quality difference between the outputs of the farms. Now the traceable farm is in a disadvantageous competitive position because of the extra production cost. Following the same procedures as the scenarios above, we can derive the sub-equilibrium profits for the players in each supply chain in the following lemma.

Lemma 4. *In the scenario (A,A), the sub-equilibrium profits ($\Pi^{AA*}_{F_i}$, $\Pi^{AA*}_{V_i}$) of the players change with respect to different extra production costs and government penalties.*

(i) When the extra production cost for the traceable farm is small, i.e., $0<c\le\frac{9(2-\sqrt{3})}{2\psi}$, we have the following three cases.

(i-a) If $0<S\le\tilde{\pi}^{AA}_{V_n}$, the sub-equilibrium profits for the players in each supply chain are:

$$\tilde{\Pi}^{AA}_{F_n}=\tilde{\pi}^{AA}_{F_n}=\frac{(9+2\psi c)^2}{108\psi},\quad \tilde{\Pi}^{AA}_{V_n}=\tilde{\pi}^{AA}_{V_n}-S=\frac{(9+2\psi c)^2}{324\psi}-S>0. \tag{20}$$

$$\tilde{\Pi}^{AA}_{F_t}=\tilde{\pi}^{AA}_{F_t}-S=\frac{(9-2\psi c)^2}{108\psi}-S>0,\quad \tilde{\Pi}^{AA}_{V_t}=\tilde{\pi}^{AA}_{V_t}=\frac{(9-2\psi c)^2}{324\psi}. \tag{21}$$

(i-b) If $\tilde{\pi}^{AA}_{V_n}<S\le\frac{3(2-\sqrt{3})}{2\psi}$, the players' sub-equilibrium profits are as follows,

$$\bar{\Pi}^{AA}_{F_n}=\frac{\sqrt{S}(9+2\psi c)-12S\sqrt{\psi}}{2\sqrt{\psi}},\quad \bar{\Pi}^{AA}_{V_n}=0. \tag{22}$$

$$\bar{\Pi}^{AA}_{F_t}=\frac{3(\sqrt{S\psi}-1)^2}{\psi}-S>0,\quad \bar{\Pi}^{AA}_{V_t}=\frac{(\sqrt{S\psi}-1)^2}{\psi}. \tag{23}$$

(i-c) *If* $S > \frac{3(2-\sqrt{3})}{2\psi}$*, the government penalty is so high that there is no trade in the non-traceable supply chain, so* $\hat{\Pi}_{F_n}^{AA} = 0$ *and* $\hat{\Pi}_{V_n}^{AA} = 0$*;* $\hat{\Pi}_{F_t}^{AA} = 0$ *and* $\hat{\Pi}_{V_t}^{AA} = 0$*.*

(ii) *When the extra production cost for the traceable farm is relatively large, i.e.,* $\frac{9(2-\sqrt{3})}{2\psi} < c \le \frac{9}{2\psi}$*, we have the following two cases.*

(ii-a) *If* $0 < S \le \tilde{\pi}_{F_t}^{AA}$*, the sub-equilibrium profits of players are the same as case (i-a).*

(ii-b) *If* $S > \tilde{\pi}_{F_t}^{AA}$*, the government penalty is so high that there is no trade in the traceable supply chain, so the same case as (i-c).*

In this scenario, the traceable farm and the non-traceable vendor are punished by the government if their sampled products are detected adulterated, and their profits decrease with the government penalty. Since the average output quality of each supply chain is the same in this scenario, the traceable supply chain is less competitive for extra production costs. However, within each supply chain, the farm holds a stronger position and occupies a more significant portion of the total revenue, as discussed at the end of §4.1.1. Therefore, when the extra production cost is relatively low (Case (i) of Lemma 4, $0 < c \le \frac{9(2-\sqrt{3})}{2\psi}$), the optimal revenue of the non-traceable vendor is smaller than that of the traceable farm (i.e., $\tilde{\pi}_{V_n}^{AA} < \tilde{\pi}_{F_t}^{AA}$). Compared to the traceable farm, the sub-equilibrium profit of the non-traceable vendor will first reduce to 0 as the government penalty increases. This case resembles the scenario (U,A) in Lemma 2. First, when the government penalty is low (case (i-a)), the farms set revenue-maximizing wholesale prices. Then, the non-traceable farm distorts the wholesale price to compensate the non-traceable vendor and keeps her continuing to source from the upstream farm (but with 0 profit). Both farms' wholesale prices reduce with higher government penalties, and until finally, the traceable farm is driven out of the market when $S > \frac{3(2-\sqrt{3})}{2\psi}$, i.e., case (i-c).

When the extra production cost is relatively large (Case (ii) of Lemma 4), the competitive disadvantage of the traceable supply chain is so significant that the traceable farm's revenue is smaller than that of the non-traceable vendor. Therefore, the direct government penalty imposed on the traceable farm would drive him out of the market when $S > \tilde{\pi}_{F_t}^{AA}$, before the non-traceable vendor considers sourcing from the outside option. This case is relatively simple, so we focus on the impact of government penalty on the farms' adulteration strategies when the extra production cost is low, given the opposing farm adulterating in the following lemma.

Lemma 5. *When the extra production cost for the traceable farm is low, i.e.,* $0 < c \le \frac{9(2-\sqrt{3})}{2\psi}$*, the farm's adulteration strategy is as follows, given the other farm adulterating.*

(i) *When* $0 < S \le min(S_{F_n}^A, \frac{3(2-\sqrt{3})}{2\psi})$*, the non-traceable farm adulterates, given the traceable farm adulterating (i.e.,* $\Pi_{F_n}^{AA*} \ge \Pi_{F_n}^{AU*}$*); otherwise, the non-traceable farm does not adulterate when* $S > min(S_{F_n}^A, \frac{3(2-\sqrt{3})}{2\psi})$*, where* $S_{F_n}^A$ *is the government penalty threshold satisfying* $\bar{\Pi}_{F_n}^{AA} = \Pi_{F_n}^{AU*}$*, and* $S_{F_n}^A > \tilde{\pi}_{V_n}^{AA}$*.*

(ii) The traceable farm's adulteration strategy, given the non-traceable farm adulterating, alters as follows.

(ii-a) When $0 < S \leq \tilde{\pi}_{V_n}^{AA}$*, the traceable farm adulterates (i.e.,* $\tilde{\Pi}_{F_t}^{AA} \geq \tilde{\Pi}_{F_t}^{UA}$*) if* $0 < S \leq min(S_{F_t}^{A1}, \tilde{\pi}_{V_n}^{AA})$*; otherwise, the traceable farm does not adulterate if* $min(S_{F_t}^{A1}, \tilde{\pi}_{V_n}^{AA}) < S \leq \tilde{\pi}_{V_n}^{AA}$*, where* $S_{F_t}^{A1} = \tilde{\pi}_{F_t}^{AA} - \tilde{\pi}_{F_t}^{UA} = \frac{r\phi(9-r\phi-2c\psi)}{27\psi}$*.*

(ii-b) When $\tilde{\pi}_{V_n}^{AA} < S \leq min(\tilde{\pi}_{V_n}^{UA}, \frac{3(2-\sqrt{3})}{2\psi})$*, the traceable farm adulterates (i.e.,* $\bar{\Pi}_{F_t}^{AA} \geq \tilde{\Pi}_{F_t}^{UA}$*) if* $\tilde{\pi}_{V_n}^{AA} < S \leq max(\tilde{\pi}_{V_n}^{AA}, S_{F_t}^{A2})$*; otherwise, the traceable farm does not adulterate if* $max(\tilde{\pi}_{V_n}^{AA}, S_{F_t}^{A2}) < S \leq min(\tilde{\pi}_{V_n}^{UA}, \frac{3(2-\sqrt{3})}{2\psi})$*, where* $S_{F_t}^{A2}$ *is the government penalty threshold satisfying* $\bar{\Pi}_{F_t}^{AA} = \tilde{\Pi}_{F_t}^{UA}$*, and* $0 < S_{F_t}^{A2} < min(\tilde{\pi}_{V_n}^{UA}, \frac{3(2-\sqrt{3})}{2\psi})$*.*

(ii-c) When $S > min(\tilde{\pi}_{V_n}^{UA}, \frac{3(2-\sqrt{3})}{2\psi})$*, the traceable farm does not adulterate (i.e.,* $\bar{\Pi}_{F_t}^{AA} < \bar{\Pi}_{F_t}^{UA}$*).*

Lemma 5 shows the synergy effect of indirect and direct penalty mechanisms in deterring adulteration. The adulteration-deterring mechanism in the non-traceable supply chain works because the farm needs to ensure the sourcing of the vendor. When the government penalty is high, the compensation loss to keep the non-traceable vendor in the market overcomes the benefit of adulteration for the non-traceable farm, which is similar to Lemma 3. In the other supply chain, the direct penalty works once the traceable farm adulterates for any government penalty, while the side-effect from the adulterating non-traceable farm varies in different ranges of government penalties. Specifically, when the government penalty is small (Part (ii)-a of Lemma 5), no side-effect takes effect on the adulteration deterring, and then only the side-effect of Scenario (A, A) works as $S > \tilde{\pi}_{V_n}^{AA}$ (Part (ii)-b). Finally, when the government penalty is large (Part (ii)-c), the side-effect from adulterating non-traceable farm works in both scenarios (U, A) and (A, A). In summary, these indirect and direct adulteration-deterring mechanisms work on the entire game, and formulate the equilibrium adulteration strategies of the farms, as we will discuss in detail in the following subsection.

4.2. Adulteration Equilibrium

In this subsection, we first derive the Stage I equilibrium adulteration strategies of farms based on the above Stage II-IV sub-equilibrium analysis. We then investigate the driving forces of traceable farm's adulteration behavior as the government penalty increases and the impact of market parameters on the equilibrium (A, U).

To answer our primary research question: How does the government penalty affect traceable farm's adulteration behavior in co-existing traceable and non-traceable supply chains? We demonstrate how the different equilibrium regions are positioned with respect to the government penalty. Specifically, thresholds of government penalty partition the parameter region into different farm adulteration equilibrium segments, as the following theorem shows.

Theorem 1. *The equilibrium adulteration strategies for traceable and non-traceable farms are*

$$(x_t^*, x_n^*) = \begin{cases} (A,A) \ if \ 0 < S \le min(S_{F_t}^{A1}, S_{F_t}^{A2}), & (24a) \\ (U,A) \ if \ min(S_{F_t}^{A1}, S_{F_t}^{A2}) < S \le min(S_{F_n}^{U}, \frac{1}{\psi}), & (24b) \\ (A,U) \ if \ min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi}) < S \le S_{F_t}^{U}, & (24c) \\ (U,U) \ if \ S > max(min(S_{F_n}^{U}, \frac{1}{\psi}), S_{F_t}^{U}), & (24d) \end{cases}$$

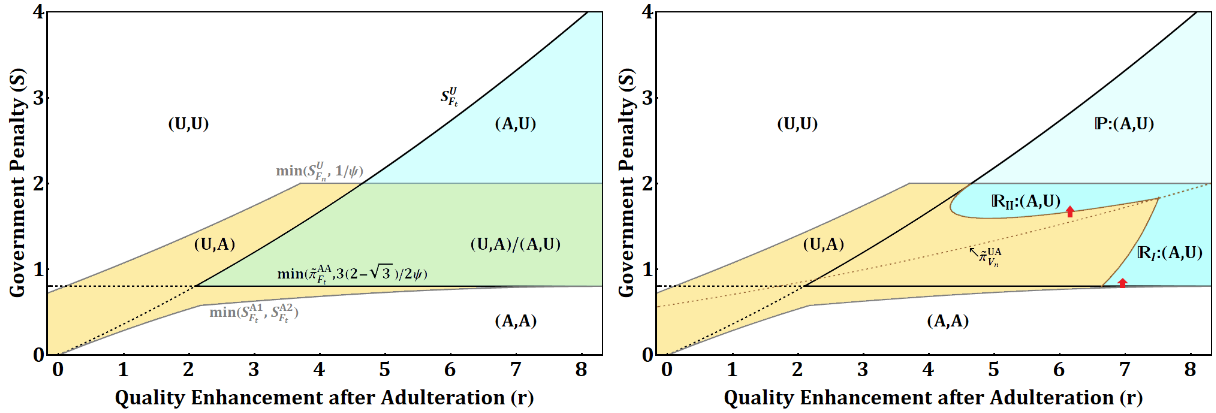
where the thresholds of government penalty are defined in §4.1.

Theorem 1 shows that, as the government penalty increases, the resulting adulteration equilibrium changes from both farms adulterate (A, A) when the penalty is low, to either the non-traceable farm adulterates (U, A) or the traceable farm adulterates (A, U), to finally both farms unadulterate (U, U) when the penalty reaches a high level. First, it is straightforward to see when the government penalty is low (case (24a)), both farms choose to adulterate. The direct penalty on the traceable farm is insignificant and the indirect penalty for the non-traceable farm has yet to take effect. Additionally, if the competitor engages in adulteration, each farm is incentivized to do the same in order to avoid being at a competitive disadvantage. Conversely, it is foreseeable that when the government penalty is exceedingly high, neither farm would choose to adulterate (case (24d)). The direct penalty for the traceable farm is so substantial that it deters adulteration, even though it could potentially widen the quality gap and enhance his competitive advantage. As for the non-traceable farm, the equilibrium wholesale price becomes distorted due to the high indirect penalty, making adulteration an unattractive option.

Between thresholds $min(S_{F_t}^{A1}, S_{F_t}^{A2})$ and $min(S_{F_n}^{U}, \frac{1}{\psi})$, the traceable farm unadulterates and the non-traceable farm adulterates (i.e., Equilibrium (U, A) in case (24b)). This scenario is to be expected, where the government inspection fails in the non-traceable supply chain. Specifically, the indirect penalty on the non-traceable farm due to distorted wholesale price either has not yet taken effect or can be offset by the profit gain from adulteration. Equilibrium (U, A) always exists since $S_{F_t}^{A2} < \tilde{\pi}_{V_n}^{UA} < S_{F_n}^{U}$ and $S_{F_t}^{A2} < \frac{3(2-\sqrt{3})}{2\psi} < \frac{1}{\psi}$ (refer to the proof of Theorem 1). On the other hand, for the equilibrium in which only the traceable farm engages in adulteration (A, U), this region might be vacant, as $min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi})$ could potentially exceed $S_{F_t}^{U}$ under specific market conditions. Nevertheless, within the region of equilibrium (A, U) $(min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi}) < S \le S_{F_t}^{U})$, the adulteration payoff due to a large quality gap for the traceable farm surpasses the direct penalty imposed by the government. As a result, government penalty proves ineffective in deterring adulteration within the traceable supply chain.

Both (A, U) and (U, A) exist in the region defined by $max(min(S_{F_t}^{A1}, S_{F_t}^{A2}), min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi})) < S \le min(S_{F_t}^{U}, S_{F_n}^{U}, \frac{1}{\psi})$ (if it exists, the green region in Figure 3(a)). To solve the issue of multi-equilibrium, we follow the refinement concept of risk dominance developed by Harsanyi and Selten (see Harsanyi and Selten 1988, Harsanyi 1995) to find the condition under which (A, U) or (U, A) is

Figure 3: One Illustration of Farms' Equilibrium Adulteration Strategies (Color Online)



(a) Before refinement (b) After refinement

Notes. The parameters are $c = 0.7$, $\Delta q = 1$, $\psi = 0.5$, $\phi = 0.5$.

selected. When each player has uncertainty on the action of the other player(s), a Nash equilibrium is considered risk dominant if either small changes of the game characteristics or the risk has no impact on the payoff dominance (Schmidt et al. 2003). A risk dominant equilibrium is more likely to occur when there is a higher degree of asymmetry in the game (Cabrales et al. 2000). This refinement method is widely employed to ensure individual player outcomes without the need for coordination with other players (Kraft et al. 2013). In our model, a unique equilibrium, either (A, U) or (U, A), survives this refinement (See Online Appendix §A.3 for more detail).

Figure 3(b) illustrates one equilibrium segmentation after risk dominance refinement with two market parameters, the quality enhancement after adulteration and the government penalty. Generally, we can see both farms prefer unadulteration to adulteration with increased government penalties, which is consistent with our conventional wisdom. Interestingly, in particular market regions, the traceable farm might switch from unadulteration to adulteration even when the government penalty increases, namely (U, A) switch to (A, U), which is indicated by two arrows in Figure 3(b). This observation is quite unexpected. Generally speaking, with the support of supply chain traceability, the source of adulterated outputs (i.e., the traceable farm) can be preciously targeted, so it would be natural that higher government penalties could reduce the adulteration occurrence rate. However, our results suggest a failure of supply chain traceability and government inspection in deterring adulteration in competing settings with both traceable and non-traceable supply chains. The reason why equilibrium (A, U) emerges with higher government penalties is different within different regions, which we will discuss in detail in the following subsection.

4.3. Driving Forces of Equilibrium (A,U) and Impact of Market Parameters

To explain why higher government penalties and supply chain traceability fail to deter adulteration and investigate the driving forces of traceable farm's adulteration behavior, we focus on the emergence and switch of the equilibrium (A,U). Figure 3(b) delineates three potential regions for equilibrium (A,U), denoted as Region $\mathbb{P}$, Region $\mathbb{R}_I$ and Region $\mathbb{R}_{II}$. Here, $\mathbb{P}(A,U)$ signifies a unique equilibrium (A,U) exists with no refinement in this region, whereas $\mathbb{R}_I$ and $\mathbb{R}_{II}$ imply those two regions survive the risk dominance refinement (Details of the refinement process and the formulation of Region $\mathbb{R}_I$ and $\mathbb{R}_{II}$ are provided in §A.3 of the Online Appendix). We will also study the impact of different market parameters on regions of equilibrium (A,U). In particular, we focus on the traceable products' quality enhancement after adulteration (r), the initial quality difference between traceable and non-traceable farms' outputs (Δq), and the traceable farm's extra production cost (c). In our model, the quality enhancement level for traceable products after adulteration is r, and this increment is $\Delta q + r$ for non-traceable products. Therefore, both farms are incentivized to adulterate if the quality enhancement is strong, and a higher initial quality difference induces the non-traceable farm to have a higher incentive to adulterate. It is straightforward to see that extra production cost denotes the traceable farm's competitive disadvantage, so higher extra production cost mitigates the quality enhancement strength after adulteration; therefore, the traceable farm has less incentive to adulterate when the extra production cost is higher, i.e., the regions of equilibrium (A,U) will shrink. In the following, we will explain the formulation of these regions of equilibrium (A,U) one by one. Specifically, we analyze the behaviors of farms' adulteration (or not) and switch of $(U,A)/(A,U)$ mainly from the perspective of the traceable farm's incentive.

i) Region $\mathbb{R}_I$ In this case, the indirect penalty on the adulterating non-traceable farm has not come into effect because $S \leq \tilde{\pi}_{V_n}^{UA}$. As we have discussed in Part (i) of Lemma 2 in §4.1.3, the government penalty on the non-traceable vendor is not that high so the outside sourcing option is not under consideration, and the penalty does not strategically impact the adulterating non-traceable farm's wholesale price. Naturally, the non-traceable farm would adulterate and the traceable farm not. However, in the (U,A) scenario, the traceable farm finds himself in an exceedingly disadvantageous competitive position due to lower output quality and the extra production cost. Therefore, under specific conditions (i.e., in Region $\mathbb{R}_I$), the traceable farm might lean towards adulteration to achieve a higher output quality (compared to the non-traceable one), even with the direct government penalty imposed on him. Therefore, equilibrium (A,U) survives the refinement and Region $\mathbb{R}_I$ exists.

As discussed above, the refined equilibrium (A,U) in region $\mathbb{R}_I$ arises from the pursuit of a stronger competitive advantage for the traceable farm, characterized by a larger quality enhancement (r) and a smaller initial quality difference (Δq). As the initial quality difference increases, Region $\mathbb{R}_I$ will monotonically shrink.

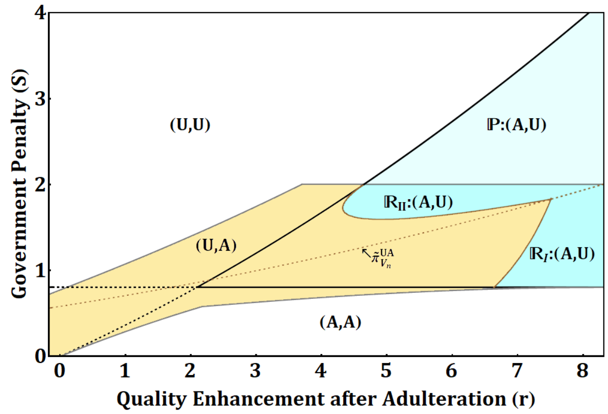
ii) Region $\mathbb{R}_{II}$ In this case, the indirect penalty on the adulterating non-traceable farm takes effect, because $S > \tilde{\pi}_{V_n}^{UA}$. As we discussed in Part (ii) of Lemma 2, the government penalty on the non-traceable vendor is so high that the adulterating non-traceable farm is compelled to deviate from the revenue-maximizing wholesale price ($\bar{w}_n < \tilde{w}_n$) to ensure continued sourcing by the non-traceable vendor. This, in turn, has a side-effect on the traceable farm, who also has to set a distorted wholesale price ($\bar{w}_t < \tilde{w}_t$) to remain competitive with the non-traceable one. Consequently, in equilibrium (U, A), even though the traceable farm refrains from adulteration, his profits experience a decline with an increase of government penalty (i.e., side-effect of indirect penalty). When the government penalty reaches a relatively high level, the side-effect of the indirect penalty becomes so pronounced that the traceable farm finds it more favorable to engage in adulteration. It induces the non-traceable farm to refrain from adulteration, so the traceable farm can avoid the side-effect of indirect penalty. This dynamic illustrates why the equilibrium (A, U) in Region $\mathbb{R}_{II}$ survives through the refinement process with an increase of government penalty.

iii) Region $\mathbb{P}$ The equilibrium (A, U) in this region is a unique equilibrium with no refinement. Similar to the dynamics observed in Region $\mathbb{R}_{II}$, the traceable farm's wholesale price is distorted when $S > \tilde{\pi}_{V_n}^{UA}$, and the side-effect of indirect penalty for the traceable farm intensifies with an increase of government penalty. In this case, the traceable farm finds himself compelled to adulterate to avoid the side-effect; failure to do so would result in zero profit due to a lack of demand when $S > \frac{1}{\psi}$ (Part (iii) of Lemma 2). In other words, if the traceable farm abstains from adulteration while the non-traceable farm engages in it, the former would be driven out of the market. Therefore, the equilibrium (A, U) in Region $\mathbb{P}$ could exist when the penalty is large.

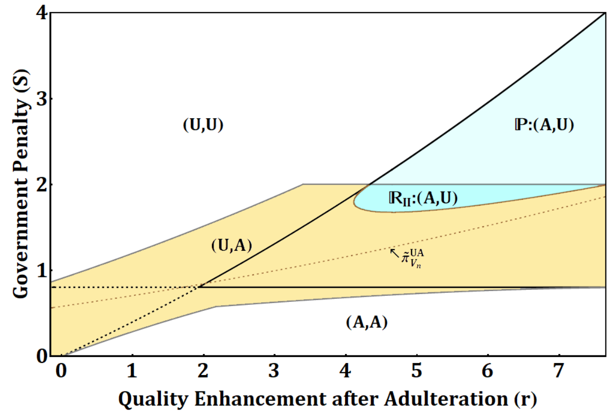
As discussed above, the refined equilibrium (A, U) in Region $\mathbb{R}_{II}$ and the equilibrium (A, U) in Region $\mathbb{P}$ are formulated due to the same driving force, so they arise or dwindle synchronously. These two regions require conditions that the extra production cost is small; otherwise, the traceable farm could not burden the direct large government penalty on him and would deviate to unadulterate. Similarly, the initial quality difference can not be large, so the adulterating incentive for the non-traceable farm is not strong.

In summary, the equilibrium (A, U) in Region $\mathbb{R}_I$ arises solely from the competitive dynamic between the two supply chains. The traceable farm chooses to adulterate and bears the penalty in exchange for a higher quality advantage, given that the government penalty has no effect on the adulterating non-traceable farm. However, the equilibrium (A, U) in Region $\mathbb{R}_{II}$ and $\mathbb{P}$ is the result of a combination of supply chain competition and government penalty. Here, the traceable farm is disinclined to unadulterating due to the weakened competitive position it would entail. Moreover, he would also bear the side-effect of the indirect penalty from the adulterating non-traceable farm if he does not adulterate. Consequently, as the government penalty increases, the traceable farm is

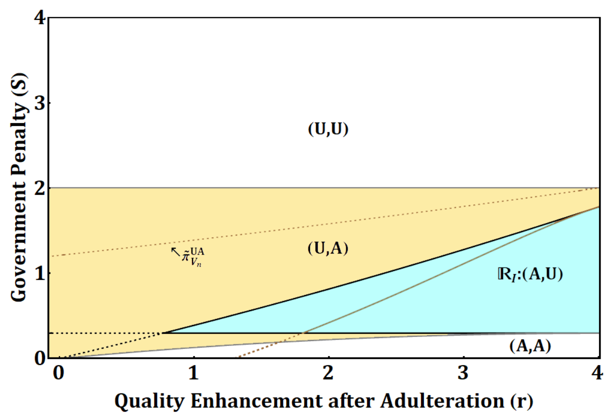
Figure 4: How Government Penalties and Quality Enhancement Impact Equilibrium (A,U) (Color Online)



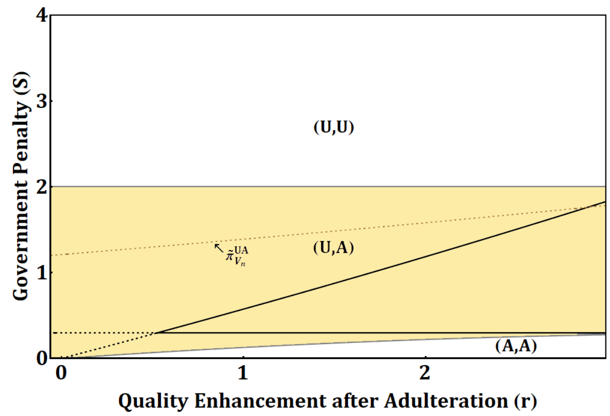
(a) With Region $\mathbb{R}_I$, $\mathbb{R}_{II}$ and $\mathbb{P}$



(b) No Region $\mathbb{R}_I$



(c) No Region $\mathbb{R}_{II}$ or $\mathbb{P}$



(d) No Region $\mathbb{R}_I$, $\mathbb{R}_{II}$ or $\mathbb{P}$

Note. The parameters are $\psi = 0.5$, $\phi = 0.5$. *In subfigure (a),* $c = 0.7$, $\Delta q = 1$; *In subfigure (b),* $c = 0.7$, $\Delta q = 2$; *In subfigure (c),* $c = 5$, $\Delta q = 6$; *In subfigure (d),* $c = 5$, $\Delta q = 11$.

compelled to engage in adulteration. The following proposition summarizes the market parameter conditions that we discussed above and characterizes the emergence conditions of three types of equilibrium (A,U).

PROPOSITION 1. *The conditions for three types of regions of equilibrium* (A,U) *with respect to the extra production cost and initial quality difference are as follows.*

(i) When the extra production cost and the initial quality difference are small, i.e., $0 < c \leq \frac{6\sqrt{6}-9}{2\psi}$ *and* $max(\frac{\psi c}{\phi}, \frac{12c^2\psi^2-108\psi c+135}{4\phi(2c\psi-9)}) < \Delta q \leq \frac{2\psi c}{\phi}$, *equilibria* (A,U) *in Region* $\mathbb{R}_I$, $\mathbb{R}_{II}$ *and* $\mathbb{P}$ *all emerge, as shown in Figure 4(a).*

(ii) When the extra production cost is small and the initial quality difference is large, i.e., $0 < c \leq \frac{6\sqrt{6}-9}{2\psi}$ *and* $\frac{2\psi c}{\phi} < \Delta q \leq \frac{6\sqrt{6}-9+2\psi c}{2\phi}$, *equilibria* (A,U) *in Region* $\mathbb{R}_{II}$ *and* $\mathbb{P}$ *emerge, as shown in Figure 4(b).*

(iii) When the extra production cost is large and the initial quality difference is small, i.e., $\frac{6\sqrt{6}-9}{2\psi} < c \le \frac{9}{2\psi}$ *and* $\frac{\psi c}{\phi} < \Delta q \le \frac{2\psi c}{\phi}$*, or both of them are relatively medium, i.e.,* $\frac{18-3\sqrt{2}}{2\psi} < c \le \frac{6\sqrt{6}-9}{2\psi}$ *and* $\frac{\psi c}{\phi} < \Delta q \le \frac{12c^2\psi^2-108\psi c+135}{4\phi(2c\psi-9)}$*, only equilibrium* (A,U) *in Region* $\mathbb{R}_I$ *emerges, as shown in Figure 4(c).*

(iv) When the extra production cost and initial quality difference are large, i.e., $\frac{6\sqrt{6}-9}{2\psi} < c \le \frac{9}{2\psi}$ *and* $\frac{2\psi c}{\phi} < \Delta q \le \frac{6\sqrt{6}-9+2\psi c}{2\phi}$*, or the initial quality difference is extremely large, i.e.,* $\frac{6\sqrt{6}-9+2\psi c}{2\phi} < \Delta q \le \frac{9+2\psi c}{2\phi}$*, none of the regions of equilibrium* (A,U) *emerges, as shown in Figure 4(d).*

It is easy to observe that equilibria (A,U) emerge only when the quality enhancement (r) is high, so the traceable farm has a strong incentive to adulterate. While the Region $\mathbb{R}_{II}$ and $\mathbb{P}$ could show up only when the extra production cost is small (part (i) and (ii) of Proposition 1), the single Region $\mathbb{R}_I$ in part (iii) could arise when the extra production cost is large. It is because in Region $\mathbb{R}_I$, the indirect penalty for the non-traceable farm has not come into effect and the refined equilibrium is just a result of competition. In general, part (iv) shows that when the extra production cost is very large, the competitive advantage of the traceable farm is weak, and there is no equilibrium (A,U). The other extreme case in part (iv) illustrates that the non-traceable farm is incentivized to adulterate due to the large initial quality difference, so (A,U) disappears for any $c > 0$.

5. Empirical Testing

In this section, we conduct an empirical analysis to validate the theoretical results in the previous section. We first describe the data and proxy the variables in §5.1. The hypotheses and logistic regression model are detailed in §5.2. Finally, we present the empirical results in §5.3.

5.1. Data Description

The data for our analysis is drawn from a product sampling test dataset published by CAMR. CAMR conducts agricultural product inspections in China's domestic market by taking samples from vendors of the local wholesale/wet market and testing them against specific quality standards. The branch of CAMR in each city randomly chooses the vendor and sample product to test, and publishes the test outcome weekly on their websites (Gao et al. 2022). We scraped the datasets as of August 2019, which contain the inspection records from Jan 2016 to Dec 2018. We select 32 prefectures in Zhejiang and Guangdong, two economy well-developed provinces in China, and focus on three categories of products, fruits, vegetables, and aquatic products, whose quality is quite sensitive to adulterant (large r). Each record of the dataset reports the inspection time, sampled product name, product category, product specification (whether the sampled product is packaged or not), mandatory inspection level (provincially or nationally), sampled product's provider (name and address, if available), sampled vendor (name and address), quality items being tested, and test outcome (pass or detected problem(s) in case of failure). To construct the independent variable

Table 2: Variables Summary

Variable Name	Variable Description	Mean	Standard Deviation	Min	Max
$ADUL$	The outcome of the sampling test (fail or pass)	0.11	0.31	/	/
$INSP\_FREQ$	Inspection frequency, a proxy of government penalty	2.25	6.27	-2.66	18.98
$NONTRACE$	Whether the product is non-traceable (yes or no)	0.36	0.48	/	/
IQD	Initial quality difference (small or large)	0.64	0.48	/	/
$COST$	Traceable farm's production cost (high=1 or low=0)	0.34	0.47	/	/
PKG	Whether the product is packaged (yes or no)	0.45	0.50	/	/
MD	Sampling test mandatory level (provincial or national)	0.26	0.44	/	/

Notes. To ease the interpretation of the intercept terms, we Mean Center the continuous variable $INSP\_FREQ$

government penalty, we also draw the prefectural-level agricultural product market data from the Statistical Yearbook of China for the corresponding period and prefectures, including the sales volume of local markets and government expenditure on inspections of each prefecture.

To study the association between the adulteration rate in supply chains and the government penalty, we need to quantify these related variables: adulteration rate, government penalty, supply chain traceability, initial quality difference, and production cost. We utilize the records in the above dataset to build metrics as proxies of the variables of our empirical model (summarized in Table 2) as follows. First, the outcome of each sampling test can be used as the proxy of adulteration or not ($ADUL$). Naturally, if the sampled product passes the test, it is unadulterated ($ADUL = 0$); otherwise, if the record is marked as "fail (with specific reason)", we treat it as adulterated ($ADUL = 1$). Additionally, if the name and address information of the sampled product's provider is available, i.e., the product's provenance can be targeted, we take it as traceable ($NONTRACE = 0$). On the contrary, if the provider's information is unavailable and the record for the product's provider is null, we take it as non-traceable ($NONTRACE = 1$).

To quantify the traceable farm's high or low production cost ($COST$), we employ the item of product category (fruits, vegetables, and aquatic products) in our sampling dataset. The production cost for aquatic products is high ($COST = H$), and low for fruits and vegetables ($COST = L$) for the following two reasons. First, processing, packing, and storing aquatic products are more costly, especially for fresh living ones. Second, the supply chains of aquatic products are more complex and dispersed than those of fruits and vegetables in China (Jin et al. 2021a), which results in higher trading and shipment costs for the provider. Next, to measure the initial quality difference between traceable and non-traceable farms' outputs ($IQD = 1$ for large and $IQD = 0$ for small), we utilize the item of product name in our sampling dataset, which classifies the sampled units as fresh or processed products. Fresh products tend to have shorter storage periods before consumption and are subject to more rigorous standards for processing and transportation conditions. They are more sensitive to manufacturing procedures such as packaging and traceability labeling. Therefore, extra production cost for the traceable farm has a stronger impact on the quality of fresh products, i.e., initial quality difference of fresh products is larger than that of processed products.

In Model Section §3.3, we construct the expected government penalty S as a product of the amount of government penalty charged per adulteration caught (failure in test) and the probability of a vendor being inspected. The penalty charged per test failure is uniform across the entire country, allowing us to use the inspection probability as a proxy for government penalties. According to Gao et al. (2022), the CAMR randomly samples products from the local market, with the number of tests conducted proportional to the sales volume of products sold in that market during each inspection, so the probability of a vendor being sampled per inspection is roughly the same across the prefectures in the two provinces, and we can further use the inspection frequency as a proxy for government penalties. Given the cost per sample test is relatively consistent, and higher inspection frequencies and larger market sizes result in greater government expenditure, we construct the metric for government inspection frequency, $INSP\_FREQ$, as the prefectural-level annual expenditure for inspection and testing ($GOVT\_EXPEND$) normalized by the prefectural-level agricultural business market sales volume ($SALES$), which is calculated as follows:

$$INSP\_FREQ = \frac{GOVT\_EXPEND}{SALES}.$$

The following control variables capture other factors related to the traceable farm's adulteration rate. First, agricultural products might be polluted during transportation. We include the item product specification, i.e., packaged ($PKG = 1$) or not ($PKG = 0$), because packaged products have less failure possibility due to logistic pollution. Second, to account for the impact of different sampling test standards, we include the test mandatory level ($MD = 1$ for the national level and $MD = 0$ for the provincial level) because the national inspection is conducted by the national CAMR, which assigns different sampling staff and follows different inspection criteria from the provincial inspection, leading to a difference in sampling results.

5.2. Hypotheses Development and Logistic Regression Model

In this subsection, we first develop a set of hypotheses regarding the association between government penalties and traceable farms' adulteration behavior based on the implications from the analytical results in Proposition 1. To exclude trivial scenarios, we assume the quality enhancement after adulteration is large and investigate the moderation effect of production costs and initial quality differences. We focus on the cases when the government penalty increases from a medium level to a relatively high level because an extremely low or high penalty rarely exists in real practices. First, part (i) and (ii) of Proposition 1 show that when the production cost is low, the traceable farm might adulterate when the government penalty is high, indicated as Region $\mathbb{R}_{II}$ and $\mathbb{P}$. When the initial quality difference is small, the traceable farm will adulterate if the government penalty is medium (Region $\mathbb{R}_I$ exists); otherwise, he will not adulterate when the initial quality difference is large (Region $\mathbb{R}_I$ does not exist). Therefore, we develop the following two hypotheses for the association of government penalties and traceable farms' adulteration rate.

Hypothesis 1a. For traceable products manufactured with low production costs, there is no association between a higher government penalty and a higher adulteration rate, when the initial quality difference is small.

Hypothesis 1b. For traceable products manufactured with low production costs, a higher government penalty is associated with a higher adulteration rate, when the initial quality difference is large.

Next, part (iii) and (iv) of Proposition 1 show that when the government penalty is medium, the traceable farm might adulterate (equilibrium (A,U)) but will not adulterate when the penalty is high, i.e., no Region $\mathbb{R}_{II}$ or $\mathbb{P}$. As such, when the production cost is high, the hypothesis we developed is as follows.

Hypothesis 2. For traceable products manufactured with high production costs, a higher government penalty is associated with a lower adulteration rate, no matter whether the initial quality difference is large or small.

Next, we build a logistic regression model to test the hypotheses developed above. Combining the classification of the variables in §5.1 with the analytical results in Proposition 1, we should expect to observe an association between the adulteration rate in traceable supply chains and government inspection frequencies across different prefectures. This association is influenced by factors such as supply chain traceability, initial quality differences, and production costs. The quality enhancement after adulteration for the three categories of products in our setting is high, which is consistent with the conditions of the hypotheses developed above. The data for the independent variables are recorded when samples are collected in the market, before test results (i.e., the dependent variable) and any subsequent penalties are announced. Specifically, traceable vendors provide comprehensive sourcing information to the inspection agency, regardless of whether the product is adulterated or not, while non-traceable vendors can not provide such information. Therefore, the issue of more sourcing information being provided in the event of a test failure does not arise. Since these hypotheses are classified based on high and low production costs, we specify the logistic regression regarding adulteration or not (the test outcome of failure or pass) for different production costs, which is as follows,

$$\begin{aligned} &logit(ADUL^i) = \alpha^i + \beta_1^i INSP\_FREQ^i + \beta_2^i NONTRACE^i + \beta_3^i IQD^i + \beta_4^i INSP\_FREQ^i \\ &\times NONTRACE^i + \beta_5^i NONTRACE^i \times IQD^i + \beta_6^i INSP\_FREQ^i \times IQD^i + \\ &\beta_7^i INSP\_FREQ^i \times NONTRACE^i \times IQD^i + controlvariables + \epsilon, \end{aligned} \tag{25}$$

where $i \in \{H, L\}$ represents the high or low production cost of the traceable farm ($COST$).

5.3. Regression Results

Table A.2 in Online Appendix A.4 summarizes the results of the above logistic regressions. Based on the implication of the coefficients of logistic regression (Angrist and Pischke 2009), one unit increment of inspection frequency is associated with an estimated unit change of the adulteration rate as follows

$$\Delta logit(ADUL^i) = \beta_1^i + \beta_4^i NONTRACE^i + \beta_6^i IQD^i + \beta_7^i NONTRACE^i \times IQD^i. \tag{26}$$

Therefore, for traceable supply chain with low initial quality difference ($NONTRACE^i = 0$ and $IQD^i = 0$), a unit change of inspection frequency is associated with β_1^i unit change of the adulteration rate; for traceable supply chain with high initial quality difference ($NONTRACE^i = 0$ and $IQD^i = 1$), a unit change of inspection frequency is associated with $\beta_1^i + \beta_6^i IQD^i$ unit change of the adulteration rate. Specifically, for the low production cost case, Panel A shows that the association between adulteration rate and inspection frequency is highly significant, and $\Delta logit(ADUL^L)$ equals $-0.021 + 0.114 = 0.093$ and -0.021 for high and low initial quality difference, respectively. The result suggests that for traceable products manufactured with low production costs, the adulteration rate is positively correlated with the government penalty (proxied by $INSP\_FREQ$) when the initial quality difference is high and negatively correlated with the government penalty when the initial quality difference is low, which supports Hypothesis 1b, but does not support Hypothesis 1a.

Similarly, for the traceable products manufactured with high production costs, Panel B shows that the association between adulteration rate and inspection frequency is significant. However, the moderation effect of the initial quality difference is not significant. The values of $\Delta logit(ADUL^H)$, according to (26), are $-0.045 - 0.008 = -0.053$ and -0.045 for high and low initial quality differences, respectively. The result suggests that for either large or small initial quality difference, the adulteration rate is negatively correlated with the government penalty when the production cost is high, which supports Hypothesis 2.

The empirical results suggest that government inspections, even with supply chain traceability, may not effectively reduce adulteration risks in competitive supply chains. Specifically, the adulteration rate is positively correlated with the inspection frequency for traceable products that have low production costs and a large initial quality gap. Along with the analytical findings in §4, we recommend that inspection agencies move beyond random sampling policies. Inspection resources should be more strategically allocated between traceable and non-traceable supply chains, and the specific characteristics of the products being sampled shall be taken into account.

6. Conclusion

There are challenges in developing and implementing sampling and inspection approaches to mitigate food safety problems with scarce government resources, especially in developing economies like China. Many might believe that downstream regular inspection supported by supply chain traceability would perform well in a decentralized regulatory system and improve the safety of food supply chain outcomes. This paper builds a game-theoretical model to study a traceable farm's adulteration behavior with respect to government penalties and other market parameters in a competing setting. We identify different driving forces behind the equilibria where the traceable farm adulterates, and the non-traceable one does not. When the government penalty is not that high, and the indirect penalty on the non-traceable farm has yet to take effect, the traceable farm chooses to adulterate to hold a better competitive position. When the government penalty is relatively high, and the indirect on the non-traceable farm comes into effect, the traceable farm prefers adulteration to avoid the severe side-effect of the indirect penalty. Hence, there exist scenarios where higher government penalties can inadvertently induce inferior behavior of the traceable farm. We conduct a preliminary empirical analysis based on a sampling test dataset of China's domestic agricultural product market, which validates the analytical results.

The above findings of our paper suggest that regular government inspection with supply chain traceability might target the adulteration source and weaken the incentive for adulteration by direct penalty, but can generate new safety issues due to supply chain competition and the side-effect of indirect penalty. Consequently, it can not fully deter adulteration in farming supply chains under a competing setting. To make the food supply chain more sustainable and protect public safety, governments must consider supply chain structure and competition, besides other risk drivers from data-driven analytics. The nuanced understanding of the driving forces of adulteration can guide policymakers when implementing traceability in a limited inspection resources situation. While our model is rooted in the farming supply chains of China, its framework can be extrapolated to yield insights for other regions or markets featuring competition between traceable and non-traceable supply chains, as well as decentralized inspection agencies.

In this paper, we develop a parsimonious model that can be extended to encompass additional facets of the complex food manufacturing and inspection system. Several vital assumptions deserve more discussions, and related research opportunities emerge. First, our model assumes the government agency randomly takes samples from the two vendors. Hence, the inspection frequency is uniform for traceable and non-traceable vendors. It would generate new insights and provide a more practical guide for the government agency if it strategically allocates the limited inspection resources to the two types of vendors and differentiates the inspection frequency. Although the current stylized model captures the crucial elements of competing supply chain structures, the farming industry in actual practice is more fragmented and opaque. Hence, another research direction could extend the one-farm, one-vendor supply chain to more complex networks, where inspection resource allocation becomes more challenging, and supply chain traceability might play a more prominent role. Finally, it is interesting to examine a situation in which each vendor is the leader in setting the wholesale price for the upstream farm. Different pricing mechanisms present an opportunity to show the impact of supply chain power structure on food adulteration and inspection systems.

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Online Appendix to "The Impact of Government Inspections on Farms' Adulteration Behaviors in Co-Existing Traceable and Non-Traceable Supply Chains"

Appendix A: Notations, Modeling Assumpstion Justifications, Risk Dominance Refinement, and Logistic Regression Results

A.1. Notations

Table A.1: Table of Notation

Subscript i	Players, profits and decisions in the corresponding supply chain. $i \in \{t, n\}$, where t represents traceable supply chain, and n represents non-traceable supply chain
Superscript x_i	Adulteration decisions of farms, and $x_i \in \{A, U\}$, where A represents adulteration and U represents unadulteration
Farm (F_i) parameters	
w_i	Wholesale price
c	Traceable farm's extra unit production cost
π_{F_i}	Revenue of farms from selling corresponding products without considering government penalty
Π_{F_i}	Profit of farms from selling corresponding products
Vendor (V_i) parameters	
p_i	Retail price
π_{V_i}	Revenue of vendors from selling corresponding products without considering government penalty
Π_{V_i}	Profit of vendors from selling corresponding products
Quality parameters	
q_j	Quality of products, and $j \in \{H, L\}$ represents high quality and low quality of products, respectively
θ_i^H	Probability of producing high-quality products if the farm does not adulterate.
θ^{max}	Probability of producing high-quality products if the farm chooses to adulterate
Q_{i0}	Expected quality of products if the farm does not adulterate
Q_A	Expected quality of adulterated products
Δq	Initial quality difference between traceable and non-traceable products
r	Quality enhancement level for traceable products after adulteration
Market and regulation parameters	
D_i	Demand of traceable/non-traceable products
ϕ	The quality-differential effect on demand
ψ	The price-differential effect on demand
S	Expected government penalty

A.2. Modeling Assumption Justifications

In this subsection, we provide some justifications for the important assumptions of our model.

Information Structure. In our model, we assume that vendors are aware of the farms' adulteration behavior before setting retail prices, following the assumption made by Dong et al. (2022). In their model, the actions of the upstream supplier are known to all players, but the presence of a food safety issue cannot be directly observed due to the inherent randomness in risky actions and output safety. In contrast, our model assumes that both the adulteration behavior and the resulting food safety issues are deterministic. Vendors, operating within a centralized wet market, have access to side information channels or can audit and observe the average quality of each other's inputs, allowing them to infer the farms' adulteration decisions. Consequently, we assume that the adulteration decisions of the farms are common knowledge among all four players.

Market Reaction. The presence of adulteration can be challenging to notice even if unsafe products make their way into the market and are subsequently purchased by consumers. Adding adulterants by farms before the quality of the outputs is realized can undoubtedly have adverse effects on consumer health. However, such adulterated products typically do not immediately result in a food safety incident, and in most cases, their impact on consumer health is relatively minor and might only become noticeable after an extended period (i.e., falling into the "gray area" as discussed in Dong et al. 2022). Therefore, consumers do not usually factor in the risk of adulteration when making a purchase decision, as it can only be detected through sampling tests conducted by the government agency.

Uniform Government Penalty. We assume that if either farm adulterates and the government agency detects it, the penalty is the same for the traceable farm and the non-traceable vendor and irrelevant to each supply chain's sales amount or revenue. This assumption lends tractability to our model but still can capture the main factors of government penalty in practice. First, the monetary penalty of the government is the staircase, i.e., a fixed amount of money is charged if the sales amount is in a certain window[6]. Second, the uniform penalty assumption also reflects the limitation of the government agency's ability to record the sale amount or revenue of the vendors in the market. In practice, the penalty is charged only based on the amount for sampling tests, which is a small part of the sales amount, so the penalty is the same for the players (SAMR 2021). Nevertheless, we release this assumption in the Online Supplement D, where the amount of government penalty is proportional to the sale amount (Mu et al. 2016, Levi et al. 2020b), and show that the main results of our base model still hold.

[6] According to the Food Safety Law of China, "Food producers or distributors who violate the Law by engaging in unauthorized food production, distributing activities or production of food additives, ... and shall be subject to a fine of RMB 50,000 - 100,000 if the total value of the food or food additive is less than RMB 10,000 or a fine between 10 and 20 times the total value of the commodity if the total value of the commodity exceeds RMB 10,000" (Clever 2015).

A.3. Risk dominance refinement

According to Theorem 1, if $min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi}) < S_{F_t}^U$, both (A,U) and (U,A) can arise as the farms' equilibrium adulteration strategies when

$$max(min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi}), min(S_{F_t}^{A1}, S_{F_t}^{A2})) < S \leq min(S_{F_n}^U, \frac{1}{\psi}, S_{F_t}^U). \tag{A.1}$$

Based on the proof of Lemma 5, we know that $\frac{3(2-\sqrt{3})}{2\psi} > S_{F_t}^{A2}$, and additionally, we can show $\tilde{\pi}_{F_t}^{AA} > S_{F_t}^{A1}$ and $S_{F_t}^{A1} > S_{F_t}^{A2}$ if $\tilde{\pi}_{F_t}^{AA} < \frac{3(2-\sqrt{3})}{2\psi}$. Hence, $min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi}) > min(S_{F_t}^{A1}, S_{F_t}^{A2})$. In addition, Lemma C3 shows $S_{F_n}^U > S_{F_t}^U$ if $S_{F_n}^U < \frac{1}{\psi}$. Hence, Equation (A.1) can be refined as follows:

$$min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi}) < S \leq min(\frac{1}{\psi}, S_{F_t}^U).$$

To resolve the issue of multiple equilibria, we follow the refinement concept of risk dominance developed by Harsanyi and Selten (See Harsanyi and Selten (1988), Harsanyi (1995)) to find the condition under which one of the two equilibria is selected. For (A,U) to risk dominate (U,A), the collective loss of deviation from (A,U) must be higher than that of deviating from (U,A). Mathematically, it is given by

$$\left(\Pi_{F_t}^{UU*} - \Pi_{F_t}^{AU*}\right)\left(\Pi_{F_n}^{AA*} - \Pi_{F_n}^{AU*}\right) \geq \left(\Pi_{F_t}^{AA*} - \Pi_{F_t}^{UA*}\right)\left(\Pi_{F_n}^{UU*} - \Pi_{F_n}^{UA*}\right). \tag{A.2}$$

Combining Lemma 2 and Lemma 4, which characterize the value of $\Pi_{F_t}^{AA*}$, $\Pi_{F_n}^{AA*}$, $\Pi_{F_t}^{UA*}$, and $\Pi_{F_n}^{UA*}$ given different government penalty, Equation (A.2) can be written as

$$(\Pi_{F_t}^{UU*} - \Pi_{F_t}^{AU*})(\hat{\Pi}_{F_n}^{AA} - \Pi_{F_n}^{AU*}) \geq (\hat{\Pi}_{F_t}^{AA} - \tilde{\Pi}_{F_t}^{UA})(\Pi_{F_n}^{UU*} - \tilde{\Pi}_{F_n}^{UA})$$

when $min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi}) < S \leq min(S_{F_t}^U, \tilde{\pi}_{V_n}^{UA})$ (Condition I); and

$$(\Pi_{F_t}^{UU*} - \Pi_{F_t}^{AU*})(\hat{\Pi}_{F_n}^{AA} - \Pi_{F_n}^{AU*}) \geq (\hat{\Pi}_{F_t}^{AA} - \bar{\Pi}_{F_t}^{UA})(\Pi_{F_n}^{UU*} - \bar{\Pi}_{F_n}^{UA})$$

when $min(S_{F_t}^U, \tilde{\pi}_{V_n}^{UA}) < S \leq min(\frac{1}{\psi}, S_{F_t}^U)$ (Condition II).

Next, we investigate the conditions for equilibrium (A,U) dominates (U,A) in the following two cases. (i) $min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi}) < S \leq min(S_{F_t}^U, \tilde{\pi}_{V_n}^{UA})$. Define

$$\begin{aligned}\tilde{R}(S) &= (\Pi_{F_t}^{UU*} - \Pi_{F_t}^{AU*})(\hat{\Pi}_{F_n}^{AA} - \Pi_{F_n}^{AU*}) - (\hat{\Pi}_{F_t}^{AA} - \tilde{\Pi}_{F_t}^{UA})(\Pi_{F_n}^{UU*} - \tilde{\Pi}_{F_n}^{UA}) \\ &= (\tfrac{(9+2\phi\Delta q-2\psi c)^2}{108\psi} - (\tfrac{(9+2\phi(\Delta q+r)-2\psi c)^2}{108\psi} - S))(0 - \tfrac{(9-2\phi(\Delta q+r)+2\psi c)^2}{108\psi}) - \\ &\quad (0 - \tfrac{(9-2\phi r-2\psi c)^2}{108\psi})(\tfrac{(9-2\phi\Delta q+2\psi c)^2}{108\psi} - \tfrac{(9+2\phi r+2\psi c)^2}{108\psi}).\end{aligned} \tag{A.3}$$

We can show $\tilde{R}(S)$ decreases in S since $\frac{\mathrm{d}\tilde{R}(S)}{\mathrm{d}S} = -\frac{(9-2\phi(\Delta q+r)+2\psi c)^2}{108\psi} < 0$. Additionally, define $\tilde{S}$ as the penalty threshold that ensures $\tilde{R}(\tilde{S}) = 0$, so we have (A,U) dominates (U,A) when $S < \tilde{S}$. Therefore, define

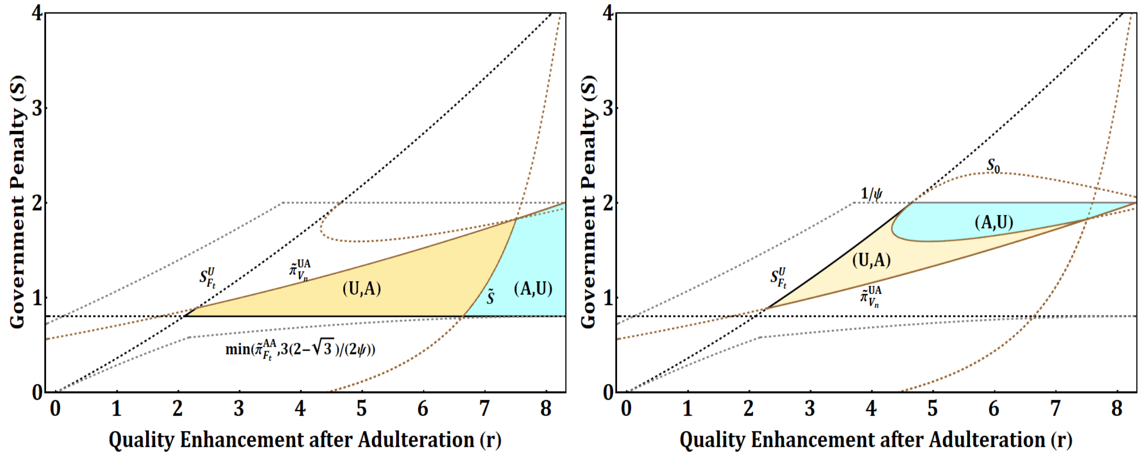
$$\mathbb{R}_I = \{S : min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi}) < S \leq min(\tilde{S}, S_{F_t}^U, \tilde{\pi}_{V_n}^{UA})\}, \tag{A.4}$$

as the region where (A,U) dominates (U,A) given Condition I (Shown in Figure A.1(a)).

(ii) $min(S_{F_t}^U, \tilde{\pi}_{V_n}^{UA}) < S \leq min(S_{F_t}^U, \frac{1}{\psi})$. Define

$$\begin{aligned}\bar{R}(S) &= \bar{R}^{AU}(S) - \bar{R}^{UA}(S) \\ &= (\Pi_{F_t}^{UU*} - \Pi_{F_t}^{AU*})(\hat{\Pi}_{F_n}^{AA} - \Pi_{F_n}^{AU*}) - (\hat{\Pi}_{F_t}^{AA} - \bar{\Pi}_{F_t}^{UA})(\Pi_{F_n}^{UU*} - \bar{\Pi}_{F_n}^{UA}) \\ &= (\tfrac{(9+2\phi\Delta q-2\psi c)^2}{108\psi} - (\tfrac{(9+2\phi(\Delta q+r)-2\psi c)^2}{108\psi} - S))(0 - \tfrac{(9-2\phi(\Delta q+r)+2\psi c)^2}{108\psi}) \\ &\quad - (0 - \tfrac{3(-1+\sqrt{\psi S})^2}{\psi})(\tfrac{(9-2\phi\Delta q+2\psi c)^2}{108\psi} - (\tfrac{\sqrt{S}(9+2\phi r+2\psi c)}{2\sqrt{\psi}} - 6S)),\end{aligned} \tag{A.5}$$

Figure A.1: Illustration of Risk Dominance Equilibrium



(a) (b)

Notes. The parameters are $c = 0.7$, $\Delta q = 1$, $\psi = 0.5$, $\phi = 0.5$.

where $\bar{R}^{UA}(S) = (\hat{\Pi}^{AA}_{F_t} - \bar{\Pi}^{UA}_{F_t})(\Pi^{UU*}_{F_n} - \bar{\Pi}^{UA}_{F_n})$ and $\bar{R}^{AU}(S) = (\Pi^{UU*}_{F_t} - \Pi^{AU*}_{F_t})(\hat{\Pi}^{AA}_{F_n} - \Pi^{AU*}_{F_n})$.

We can show that $\bar{R}(S) = 0$ has at most two solutions when $min(S^U_{F_t}, \tilde{\pi}^{UA}_{V_n}) < S \le S^U_{F_t}$. In addition, solutions of $\bar{R}(S) = 0$ (if exist) consist of a smooth simple closed curve S_0, and let $\mathcal{RD}$ be the region consisting of S_0 and its interior. When $S \in \mathcal{RD}$, $\bar{R}^{UA}(S) < \bar{R}^{AU}(S)$. Therefore, we can define

$$\mathbb{R}_{II} = \{S : \mathcal{RD} \cap min(S^U_{F_t}, \tilde{\pi}^{UA}_{V_n}) < S \le min(S^U_{F_t}, \tfrac{1}{\psi})\}, \tag{A.6}$$

as the region where (A,U) dominates (U,A) given Condition II (Shown in Figure A.1(b)).

A.4. Logistic Regression Results

The following table shows the results of our logistic regression model in Equation (25).

Table A.2: Regression Results

	(1)	(2)	(3)	(4)	(5)
		Panel A: Low production cost			
$INSP\_FREQ$	-0.011^{***}	-0.012^{***}	-0.017^{***}	-0.017^{***}	-0.021^{***}
	(0.989)	(0.988)	(0.983)	(0.983)	(0.979)
$NONTRACE$	-0.21^{*}	-0.033	-0.298	-0.484^{*}	-1.844^{***}
	(0.811)	(0.968)	(0.742)	(0.616)	(0.158)
IQD	-0.883^{***}	-0.762^{***}	-0.765^{***}	-0.742^{***}	-1.024^{***}
	(0.413)	(0.467)	(0.465)	(0.476)	(0.359)
PKG	-0.387^{***}	-0.429^{***}	-0.447^{***}	-0.455^{***}	-0.404^{***}
	(0.679)	(0.651)	(0.639)	(0.634)	(0.668)
MD	-1.041^{***}	-1.04^{***}	-1.054^{***}	-1.051^{***}	-1.075^{***}
	(0.353)	(0.353)	(0.349)	(0.349)	(0.341)
$NONTRACE * IQD$		-0.276	-0.05	0.136^{***}	1.751^{***}
		(0.758)	(0.952)	(1.145)	(5.761)
$NONTRACE * INSP\_FREQ$			0.031^{***}	0.047^{***}	0.139^{***}
			(1.032)	(1.049)	(1.149)
$IQD * INSP\_FREQ$				-0.024	0.114^{***}
				(0.976)	(1.12)
$IQD * INSP\_FREQ * NONTRACE$					-0.26^{***}
					(0.771)
R^2	0.073	0.073	0.074	0.075	0.081
Number of obs.	29226	29226	29226	29226	29226
		Panel B: High production cost			
$INSP\_FREQ$	-0.019^{***}	-0.017^{***}	-0.045^{***}	-0.047^{***}	-0.045^{***}
	(0.981)	(0.984)	(0.956)	(0.954)	(0.956)
$TRAC$	-0.402^{***}	-0.963^{***}	-1.47^{***}	-1.172^{***}	-1.124^{***}
	(0.669)	(0.382)	(0.23)	(0.31)	(0.325)
IQD	-0.164	-0.831^{***}	-0.829^{***}	-0.872^{***}	-0.831^{***}
	(0.848)	(0.435)	(0.437)	(0.418)	(0.436)
PKG	-0.559^{***}	-0.371^{***}	-0.416^{***}	-0.415^{***}	-0.411^{***}
	(0.572)	(0.69)	(0.659)	(0.66)	(0.663)
MD	-1.633^{***}	$-1,652^{***}$	-1.694^{***}	-1.7^{***}	-1.7^{***}
	(0.195)	(0.192)	(0.184)	(0.183)	(0.183)
$NONTRACE * IQD$		1.116^{***}	1.554^{***}	1.269^{***}	1.173^{***}
		(3.051)	(4.73)	(3.558)	(3.232)
$NONTRACE * INSP\_FREQ$			0.099^{***}	0.063^{***}	0.054^{***}
			(1.104)	(1.065)	(1.055)
$IQD * INSP\_FREQ$				0.056^{***}	-0.008
				(1.057)	(0.992)
$IQD * INSP\_FREQ * NONTRACE$					0.074
					(1.077)
R^2	0.093	0.097	0.109	0.11	0.111
Number of obs.	14910	14910	14910	14910	14910

Notes. numbers in the parentheses represent odds ratio. ***,**,* indicate significance at the 1%, 5%, and 10% levels, respectively.

Appendix B: Proofs of the Results

B.1. Proof of Lemma 1.

From equation (11) and (13), We can show $\Pi_{F_t}^{AU*} - \Pi_{F_t}^{UU*} = \pi_{F_t}^{AU*} - S - \pi_{F_t}^{UU*} = S_{F_t}^{U} - S$. It is obvious that $\Pi_{F_t}^{AU*} < \Pi_{F_t}^{UU*}$ when $S > S_{F_t}^{U}$. $\square$

B.2. Proof of Lemma 2.

We first use backward induction to derive the optimal revenue of players ($\pi_{V_i}^{UA}$ and $\pi_{F_i}^{UA}$) by assuming vendors always procure from the upstream farm, and then we consider the effect of government penalty on the characterization of the equilibrium.

Given the wholesale prices, it is straightforward to show the concavity of each vendor's revenue $\pi_{V_i}^{UA}$ on the retail price p_i, $i \in \{t,n\}$, and the optimal prices for vendors are

$$p_t = \frac{3-2\phi r+\psi(4w_t+2w_n)}{6\psi}, \quad p_n = \frac{3+2\phi r+\psi(2w_t+4w_n)}{6\psi}.$$

Anticipating vendors' retail prices, farms set the wholesale prices, respectively, to maximize

$$\pi_{F_t}^{UA} = \frac{(w_t-c)(3-2\phi r-2\psi(w_t-w_n))}{6}, \quad \pi_{F_n}^{UA} = \frac{w_n(3+2\phi r+2\psi(w_t-w_n))}{6}.$$

It is obvious that farms' revenue is concave in w_i. Hence, we can get the revenue-maximizing wholesale prices:

$$\tilde{w}_t = \frac{9-2\phi r+4\psi c}{6\psi}, \quad \tilde{w}_n = \frac{9+2\phi r+2\psi c}{6\psi}, \tag{B.1}$$

and the corresponding optimal revenues of the players are

$$\tilde{\pi}_{F_n}^{UA} = \frac{(9+2\phi r+2\psi c)^2}{108\psi}, \quad \tilde{\pi}_{V_n}^{UA} = \frac{(9+2\phi r+2\psi c)^2}{324\psi}.$$
$$\tilde{\pi}_{F_t}^{UA} = \frac{(9-2\phi r-2\psi c)^2}{108\psi}, \quad \tilde{\pi}_{V_t}^{UA} = \frac{(9-2\phi r-2\psi c)^2}{324\psi}.$$

Then we consider the effect of government penalty on the equilibrium. Because $\Pi_{V_n}^{UA} = \pi_{V_n}^{UA} - S$, the non-traceable vendor (V_n) might not procure from the non-traceable farm based on the government penalty.

(i) When $0 < S \le \tilde{\pi}_{V_n}^{UA}$, V_n would not leave the market because its equilibrium profit $\Pi_{V_n}^{UA*} = \tilde{\pi}_{V_n}^{UA} - S > 0$ and farms set the revenue-maximizing wholesale prices as shown in Equation (B.1). Plug the revenue-maximizing wholesale prices and retail prices into players' profit functions, we can get profits shown in Lemma 2-(i).

(ii) When $\tilde{\pi}_{V_n}^{UA} < S \le min(\frac{(9+2\phi r+2\psi c)^2}{144\psi}, \frac{1}{\psi})$, V_n will not procure from F_n if F_n still sets the revenue-maximizing wholesale price, which results in $\Pi_{F_n}^{UA} = 0$. Therefore, to keep V_n in the trade with F_n, F_n sets the wholesale price so that $\Pi_{V_n}^{UA} = \pi_{V_n}^{UA} - S = \frac{(3+2\phi r+2\psi(w_t-w_n))^2}{36\psi} - S = 0$, i.e., [7]

$$w_n = \frac{3+2\phi r-6\sqrt{S\psi}+2w_t\psi}{2\psi}. \tag{B.2}$$

Correspondingly, the traceable farm sets the wholesale price as

$$w_t = argmax_{w_t} \Pi_{F_t}^{UA} = \frac{3-2\phi r+2\psi c+2\psi w_n}{4\psi}. \tag{B.3}$$

Taking (B.2) and (B.3) together, we can get the equilibrium wholesale prices $\bar{w}_n = \frac{9+2\phi r+2\psi c-12\sqrt{S\psi}}{2\psi}$ and $\bar{w}_t = \frac{\psi c+3-3\sqrt{S\psi}}{\psi}$, respectively. Correspondingly, the equilibrium demands are $\bar{D}_t = 1-\sqrt{S\psi}$ and $\bar{D}_n = \sqrt{S\psi}$, respectively. To guarantee $\bar{w}_n \ge 0$ and $\bar{w}_t \ge c$, we need $S \le min(\frac{(9+2\phi r+2\psi c)^2}{144\psi}, \frac{1}{\psi})$, and to ensure $\bar{D}_t, \bar{D}_n \ge 0$, we need $S \le \frac{1}{\psi}$. Therefore, we need the condition $S \le min(\frac{(9+2\phi r+2\psi c)^2}{144\psi}, \frac{1}{\psi})$ for this case.

(iii) When $S > min(\frac{(9+2\phi r+2\psi c)^2}{144\psi}, \frac{1}{\psi})$, either the demand of the traceable supply chain is negative or the wholesale price of the non-traceable product is negative, so there will be no trade in our setting. $\square$

[7] The other solution $w_n = \frac{3+2\phi r+6\sqrt{S\psi}+2w_t\psi}{2\psi}$ does not fit our setting.

B.3. Proof of Lemma 3.

We show the threshold of government penalty for the non-traceable farm's adulteration strategy by comparing the sub-equilibrium profit of F_n in scenarios (U,A) and (U,U), i.e., $\Pi_{F_n}^{UA*}$ and $\Pi_{F_n}^{UU*}$. As shown in Equation (10), $\Pi_{F_n}^{UU*} = \frac{(9-2\phi\Delta q+2c\psi)^2}{108\psi}$, and according to Lemma 2, $\Pi_{F_n}^{UA*}$ varies with S in different ranges. First, when $0 < S \le \tilde{\pi}_{V_n}^{UA}$, it is easy to show $\Pi_{F_n}^{UA*} - \Pi_{F_n}^{UU*} = \tilde{\Pi}_{F_n}^{UA} - \Pi_{F_n}^{UU*} > 0$ always holds.

When $\tilde{\pi}_{V_n}^{UA} < S \le min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi})$, $\Pi_{F_n}^{UA*} = \bar{\Pi}_{F_n}^{UA} = \frac{\sqrt{S}(9+2\phi r+2\psi c)-12S\sqrt{\psi}}{2\sqrt{\psi}}$. Let

$$\Lambda_{F_n}^{U}(S) = \Pi_{F_n}^{UA*} - \Pi_{F_n}^{UU*} = \frac{\sqrt{S}(9+2\phi r+2\psi c)-12S\sqrt{\psi}}{2\sqrt{\psi}} - \frac{(9-2\phi\Delta q+2c\psi)^2}{108\psi}.$$

We can show $\Lambda_{F_n}^{U}(S)$ is concave in S since $\frac{\partial^2\Lambda_{F_n}^{U}}{\partial S^2} = -\frac{9+2\phi r+2\psi c}{8S^{3/2}\sqrt{\psi}} < 0$, and $\frac{\partial\Lambda_{F_n}^{U}}{\partial S} = 0$ at $S = \frac{(9+2\phi r+2\psi c)^2}{576\psi} < \tilde{\pi}_{V_n}^{UA}$. Hence, when $\tilde{\pi}_{V_n}^{UA} < S \le min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi})$, $\Lambda_{F_n}^{U}(S)$ decreases in S. When $S = \tilde{\pi}_{V_n}^{UA}$,

$$\Lambda_{F_n}^{U}(\tilde{\pi}_{V_n}^{UA}) = \frac{(\Delta q+r)\phi(9+2\psi c+\phi(r-\Delta q))}{27\psi} > 0.$$

We also have

$$\lim_{S\to+\infty} \Lambda_{F_n}^{U}(S) \to -\infty.$$

Therefore, there must exist a unique $S_{F_n}^{U} > \tilde{\pi}_{V_n}^{UA}$, such that $\Lambda_{F_n}^{U}(S_{F_n}^{U}) = 0$. Additionally, we have $S_{F_n}^{U} < \frac{(9+2\phi r+2\psi c)^2}{144\psi}$, because $\Lambda_{F_n}^{U}(\frac{(9+2\phi r+2\psi c)^2}{144\psi}) = -\frac{(9-2\phi\Delta q+2\psi c)^2}{108\psi} < 0$. Therefore, when $\tilde{\pi}_{V_n}^{UA} < S \le min(S_{F_n}^{U}, \frac{1}{\psi})$, $\Pi_{F_n}^{UA*} \ge \Pi_{F_n}^{UU*}$; when $min(S_{F_n}^{U}, \frac{1}{\psi}) < S \le min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi})$, $\Pi_{F_n}^{UA*} < \Pi_{F_n}^{UU*}$.

When $S > min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi})$, it is obvious that the non-traceable farm will not adulterate as $\Pi_{F_n}^{UA*} = 0$. In summary, the non-traceable farm will adulterate when $0 < S \le min(S_{F_n}^{U}, \frac{1}{\psi})$, otherwise, not adulterate, given the traceable farm unadulterating. □

B.4. Proof of Lemma 4.

First, by assuming there exists trade between the farm and the vendor in each supply chain, we can derive the optimal revenue of the non-traceable vendor V_n and the traceable farm F_t by backward induction as $\tilde{\pi}_{V_n}^{AA} = \frac{(9+2\psi c)^2}{324\psi}$, $\tilde{\pi}_{F_t}^{AA} = \frac{(9-2\psi c)^2}{108\psi}$. The solving process is similar to the proof of Lemma 2 thus omitted.

In this scenario, V_n and F_t get the uniform government penalty S, and their profit will reduce to 0 with the increase of S, so either V_n or F_t will have an incentive to leave the market. It is easy to see $\tilde{\pi}_{V_n}^{AA}$ increases in c, $\tilde{\pi}_{F_t}^{AA}$ decreases in c when $c > 0$, and $\tilde{\pi}_{V_n}^{AA} = \tilde{\pi}_{F_t}^{AA}$ when $c = \frac{9(2-\sqrt{3})}{2\psi}$. Therefore, we prove the Lemma in the following two ranges of c.

(i) When $0 < c \le \frac{9(2-\sqrt{3})}{2\psi}$, $\tilde{\pi}_{V_n}^{AA} \le \tilde{\pi}_{F_t}^{AA}$, so $\tilde{\pi}_{V_n}^{AA}$ will first drop to 0 with the increase of S, ahead of $\tilde{\pi}_{F_t}^{AA}$.

If $0 < S \le \tilde{\pi}_{V_n}^{AA}$, neither the F_t nor V_n would leave the market because their equilibrium profits $\Pi_{V_n}^{AA*} = \tilde{\pi}_{V_n}^{AA} - S > 0$, $\Pi_{F_t}^{AA*} = \tilde{\pi}_{F_t}^{AA} - S > 0$ if farms set the revenue-maximizing wholesale prices. Consequently, we can get the profits in Lemma 4-(i) by plugging the revenue-maximizing wholesale prices and retail prices into the profits.

If $\tilde{\pi}_{V_n}^{AA} < S \le \frac{3(2-\sqrt{3})}{2\psi}$, V_n will not procure from F_n if F_n still sets the revenue-maximizing wholesale price, which results $\Pi_{F_n}^{AA} = 0$. Therefore, to keep V_n in the trade with F_n, F_n sets the wholesale price so that $\Pi_{V_n}^{AA} = \pi_{V_n}^{AA} - S = \frac{(3+2\psi(w_t-w_n))^2}{36\psi} - S = 0$, i.e., [8]

$$w_n = \frac{3-6\sqrt{S\psi}+2w_t\psi}{2\psi}. \tag{B.4}$$

[8] The other solution $w_n = \frac{3+6\sqrt{S\psi}+2w_t\psi}{2\psi}$ does not fit our setting.

Correspondingly, the traceable farm sets the wholesale price as

$$w_t = argmax_{w_t}\Pi_{F_t}^{AA} = \frac{3+2\psi c+2\psi w_n}{4\psi}. \tag{B.5}$$

Taking (B.4) and (B.5) together, we can get the equilibrium wholesale prices $\bar{w}_n = \frac{9+2\psi c-12\sqrt{S\psi}}{2\psi}$ and $\bar{w}_t = \frac{3+\psi c-3\sqrt{S\psi}}{\psi}$, respectively. Consequently, the equilibrium demands are $\bar{D}_t = 1-\sqrt{S\psi}$ and $\bar{D}_n = \sqrt{S\psi}$, respectively. To guarantee $\bar{w}_t \geq c$ and $\bar{w}_n \geq 0$, we need $S \leq min(\frac{(9+2\psi c)^2}{144\psi}, \frac{1}{\psi})$, and to ensure $\bar{D}_t, \bar{D}_n \geq 0$, we need $S \leq \frac{1}{\psi}$. We also note $\bar{\Pi}_{F_t}^{AA}$ decreases in S, and $\bar{\Pi}_{F_t}^{AA} = 0$ when $S = \frac{3(2-\sqrt{3})}{2\psi}$. Because $\frac{3(2-\sqrt{3})}{2\psi} < \frac{(9+2\psi c)^2}{144\psi}$ and $\frac{3(2-\sqrt{3})}{2\psi} < \frac{1}{\psi}$, we need the condition $S \leq \frac{3(2-\sqrt{3})}{2\psi}$ for this case (It is a proper upper bound because $\tilde{\pi}_{V_n}^{AA} < \frac{3(2-\sqrt{3})}{2\psi}$ when $0 < c \leq \frac{9(2-\sqrt{3})}{2\psi}$).

If $S > \frac{3(2-\sqrt{3})}{2\psi}$, the traceable farm will not trade with the traceable vendor because $\Pi_{F_t}^{AA} < 0$ in this case.

(ii) When $c > \frac{9(2-\sqrt{3})}{2\psi}$, $\tilde{\pi}_{V_n}^{AA} > \tilde{\pi}_{F_t}^{AA}$, so $\tilde{\pi}_{F_t}^{AA}$ will first drop to 0 with the increase of S, ahead of $\tilde{\pi}_{V_n}^{AA}$.

If $0 < S \leq \tilde{\pi}_{F_t}^{AA}$, neither the F_t nor V_n would leave the market because their equilibrium profits $\Pi_{V_n}^{AA*} = \tilde{\pi}_{V_n}^{AA} - S > 0$, $\Pi_{F_t}^{AA*} = \tilde{\pi}_{F_t}^{AA} - S > 0$ if the farms set the revenue-maximizing wholesale prices.

If $S > \tilde{\pi}_{F_t}^{AA}$, the traceable farm will not trade with the traceable vendor because $\Pi_{F_t}^{AA} < 0$. $\square$

B.5. Proof of Lemma 5.

We show the threshold of government penalty for the non-traceable farm's adulteration strategy by comparing the sub-equilibrium profit of F_n in scenarios (A,A) and (A,U), i.e., $\Pi_{F_n}^{AA*}$ and $\Pi_{F_n}^{AU*}$. As shown in Equation (12), $\Pi_{F_n}^{AU*} = \frac{(9-2\phi(\Delta q+r)+2\psi c)^2}{108\psi}$, and according to Lemma 4, $\Pi_{F_n}^{AA*}$ varies with S in different ranges. First, when $0 < S \leq \tilde{\pi}_{V_n}^{AA}$, it is easy to show $\Pi_{F_n}^{AA*} - \Pi_{F_n}^{AU*} = \tilde{\Pi}_{F_n}^{AA} - \Pi_{F_n}^{AU*} > 0$ always holds.

When $\tilde{\pi}_{V_n}^{AA} < S \leq \frac{3(2-\sqrt{3})}{2\psi}$, $\Pi_{F_n}^{AA*} = \bar{\Pi}_{F_n}^{AA}$. Let

$$\Lambda_{F_n}^{A}(S) = \bar{\Pi}_{F_n}^{AA} - \Pi_{F_n}^{AU*} = \frac{\sqrt{S}(9+2\psi c)-12S\sqrt{\psi}}{2\sqrt{\psi}} - \frac{(9-2\phi(\Delta q+r)+2\psi c)^2}{108\psi}.$$

We can show $\Lambda_{F_n}^{A}(S)$ is concave in S since $\frac{\partial^2\Lambda_{F_n}^{A}(S)}{\partial S^2} = -\frac{9+2\psi c}{8S^{3/2}\sqrt{\psi}} < 0$, and $\frac{\partial\Lambda_{F_n}^{A}(S)}{\partial S} = 0$ at $S = \frac{(9+2\psi c)^2}{576\psi} < \tilde{\pi}_{V_n}^{AA}$. Therefore, when $\tilde{\pi}_{V_n}^{AA} < S \leq \frac{3(2-\sqrt{3})}{2\psi}$, $\Lambda_{F_n}^{A}(S)$ decreases with S. When $S = \tilde{\pi}_{V_n}^{AA}$,

$$\Lambda_{F_n}^{A}(\tilde{\pi}_{V_n}^{AA}) = \frac{\phi(\Delta q+r)(9-\phi(\Delta q+r)+2\psi c)}{27\psi} > 0.$$

We also have

$$\lim_{S\to+\infty}\Lambda_{F_n}^{A}(S) \to -\infty.$$

Therefore, there must exist a unique $S_{F_n}^{A} > \tilde{\pi}_{V_n}^{AA}$, such that $\Lambda_{F_n}^{A}(S_{F_n}^{A}) = 0$. When $\tilde{\pi}_{V_n}^{AA} < S \leq min(S_{F_n}^{A}, \frac{3(2-\sqrt{3})}{2\psi})$, $\Pi_{F_n}^{AA*} \geq \Pi_{F_n}^{AU*}$; when $min(S_{F_n}^{A}, \frac{3(2-\sqrt{3})}{2\psi}) < S \leq \frac{3(2-\sqrt{3})}{2\psi}$, $\Pi_{F_n}^{AA*} < \Pi_{F_n}^{AU*}$.

When $S > \frac{3(2-\sqrt{3})}{2\psi}$, it is easy to show $\Pi_{F_n}^{AA*} = 0 < \Pi_{F_n}^{AU*}$. In summary, the non-traceable farm will adulterate when $0 < S \leq min(S_{F_n}^{A}, \frac{3(2-\sqrt{3})}{2\psi})$, and not adulterate when $S > min(S_{F_n}^{A}, \frac{3(2-\sqrt{3})}{2\psi})$, given traceable farm adulterating.

In what follows, we show thresholds of government penalty for the traceable farm's adulteration strategy by comparing the equilibrium profit F_t in scenarios (A,A) and (U,A), i.e., $\Pi_{F_t}^{AA*}$ and $\Pi_{F_t}^{UA*}$. According to Lemma 2 and 4, both $\Pi_{F_t}^{AA*}$ and $\Pi_{F_t}^{UA*}$ varies with different S.

When $0 < S \leq \tilde{\pi}_{V_n}^{AA}$,[9] $\Pi_{F_t}^{AA*} = \tilde{\Pi}_{F_t}^{AA} = \frac{(9-2\psi c)^2}{108\psi} - S$ and $\Pi_{F_t}^{UA*} = \tilde{\Pi}_{F_t}^{UA} = \frac{(9-2\phi r-2\psi c)^2}{108\psi}$. Let

$$S_{F_t}^{A1} = \frac{(9-2\psi c)^2}{108\psi} - \frac{(9-2\phi r-2\psi c)^2}{108\psi} = \frac{r\phi(9-r\phi-2c\psi)}{27\psi} > 0.$$

It is straightforward to see $\Pi_{F_t}^{AA*} \geq \Pi_{F_t}^{UA*}$ when $0 < S \leq min(S_{F_t}^{A1}, \tilde{\pi}_{V_n}^{AA})$, and $\Pi_{F_t}^{AA*} < \Pi_{F_t}^{UA*}$ when $min(S_{F_t}^{A1}, \tilde{\pi}_{V_n}^{AA}) < S \leq \tilde{\pi}_{V_n}^{AA}$.

When $\tilde{\pi}_{V_n}^{AA} < S \leq min(\tilde{\pi}_{V_n}^{UA}, \frac{3(2-\sqrt{3})}{2\psi})$, $\Pi_{F_t}^{AA*} = \bar{\Pi}_{F_t}^{AA} = \frac{3(\sqrt{S\psi}-1)^2}{\psi} - S$ and $\Pi_{F_t}^{UA*} = \tilde{\Pi}_{F_t}^{UA} = \frac{(9-2\phi r-2\psi c)^2}{108\psi}$. Let

$$\Lambda_{F_t}^{A2}(S) = \bar{\Pi}_{F_t}^{AA} - \tilde{\Pi}_{F_t}^{UA} = \frac{3(\sqrt{S\psi}-1)^2}{\psi} - S - \frac{(9-2\phi r-2\psi c)^2}{108\psi}. \tag{B.6}$$

We can show $\Lambda_{F_t}^{A2}(S)$ is convex in S since $\frac{\partial^2 \Lambda_{F_t}^{A2}(S)}{\partial S^2} = \frac{3}{2S^{3/2}\sqrt{\psi}} > 0$, and $\frac{\partial \Lambda_{F_t}^{A2}(S)}{\partial S} = 0$ at $S = \frac{9}{4\psi} > min(\frac{3(2-\sqrt{3})}{2\psi}, \tilde{\pi}_{V_n}^{UA})$. Hence, when $\tilde{\pi}_{V_n}^{AA} < S \leq min(\tilde{\pi}_{V_n}^{UA}, \frac{3(2-\sqrt{3})}{2\psi})$, $\Lambda_{F_t}^{A2}(S)$ decreases in S. When $S = \frac{3(2-\sqrt{3})}{2\psi}$,

$$\Lambda_{F_t}^{A2}(\frac{3(2-\sqrt{3})}{2\psi}) = -\frac{(-9+2\phi r+2\psi c)^2}{108\psi} < 0. \tag{B.7}$$

When $S = \tilde{\pi}_{V_n}^{UA}$,

$$\Lambda_{F_t}^{A2}(\tilde{\pi}_{V_n}^{UA}) = -\frac{(9+2\phi r+2\psi c)^2}{324\psi} < 0. \tag{B.8}$$

We also have

$$\lim_{S\to 0} \Lambda_{F_t}^{A2}(S) = \frac{3}{\psi} - \frac{(9-2\phi r-2\psi c)^2}{108\psi} > 0. \tag{B.9}$$

Therefore, there must exist a unique $0 < S_{F_t}^{A2} < min(\tilde{\pi}_{V_n}^{UA}, \frac{3(2-\sqrt{3})}{2\psi})$, such that $\Lambda_{F_t}^{A2}(S_{F_t}^{A2}) = 0$. When $\tilde{\pi}_{V_n}^{AA} < S \leq max(S_{F_t}^{A2}, \tilde{\pi}_{V_n}^{AA})$, $\Pi_{F_t}^{AA*} \geq \Pi_{F_t}^{UA*}$; when $max(S_{F_t}^{A2}, \tilde{\pi}_{V_n}^{AA}) < S \leq min(\tilde{\pi}_{V_n}^{UA}, \frac{3(2-\sqrt{3})}{2\psi})$, $\Pi_{F_t}^{AA*} < \Pi_{F_t}^{UA*}$. We then compare $\Pi_{F_t}^{UA*}$ and $\Pi_{F_t}^{AA*}$ in the following two cases.

(i) When $\frac{3(2-\sqrt{3})}{2\psi} \leq \tilde{\pi}_{V_n}^{UA}$, we have $\tilde{\pi}_{V_n}^{AA} < \frac{3(2-\sqrt{3})}{2\psi} \leq \tilde{\pi}_{V_n}^{UA} < \min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi})$. We will compare $\Pi_{F_t}^{UA*}$ and $\Pi_{F_t}^{AA*}$ in the following two subcases, where the above thresholds are set as boundaries of the ranges, sequentially.

- (i-a) If $\tilde{\pi}_{V_n}^{AA} < S \leq \frac{3(2-\sqrt{3})}{2\psi}$, $\Pi_{F_t}^{UA*} = \tilde{\Pi}_{F_t}^{UA} = \frac{(9-2\phi r-2\psi c)^2}{108\psi}$ and $\Pi_{F_t}^{AA*} = \bar{\Pi}_{F_t}^{AA} = \frac{3(\sqrt{S\psi}-1)^2}{\psi} - S$, and $\Pi_{F_t}^{UA*} \geq \Pi_{F_t}^{AA*}$ for $max(\tilde{\pi}_{V_n}^{AA}, S_{F_t}^{A2}) < S \leq \frac{3(2-\sqrt{3})}{2\psi}$.
- (i-b) If $S > \frac{3(2-\sqrt{3})}{2\psi}$, $\Pi_{F_t}^{AA*} = 0$, so $\Pi_{F_t}^{UA*} \geq \Pi_{F_t}^{AA*}$ always hold.

(ii) When $\frac{3(2-\sqrt{3})}{2\psi} > \tilde{\pi}_{V_n}^{UA}$, we have $\tilde{\pi}_{V_n}^{AA} < \tilde{\pi}_{V_n}^{UA} \leq \frac{3(2-\sqrt{3})}{2\psi} < \min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi})$[10]. We will compare $\Pi_{F_t}^{UA*}$ and $\Pi_{F_t}^{AA*}$ in the following three subcases, where the above thresholds are set as the boundaries of the ranges, sequentially.

- (ii-a) If $\tilde{\pi}_{V_n}^{AA} < S \leq \tilde{\pi}_{V_n}^{UA}$, $\Pi_{F_t}^{UA*} = \tilde{\Pi}_{F_t}^{UA} = \frac{(9-2\phi r-2\psi c)^2}{108\psi}$ and $\Pi_{F_t}^{AA*} = \bar{\Pi}_{F_t}^{AA} = \frac{3(\sqrt{S\psi}-1)^2}{\psi} - S$, and the condition for $\Pi_{F_t}^{UA*} \geq \Pi_{F_t}^{AA*}$ is $max(\tilde{\pi}_{V_n}^{AA}, S_{F_t}^{A2}) < S \leq \tilde{\pi}_{V_n}^{UA}$.
- (ii-b) If $\tilde{\pi}_{V_n}^{UA} < S \leq \frac{3(2-\sqrt{3})}{2\psi}$, $\Pi_{F_t}^{UA*} = \bar{\Pi}_{F_t}^{UA} = \frac{3(\sqrt{S\psi}-1)^2}{\psi}$ and $\Pi_{F_t}^{AA*} = \bar{\Pi}_{F_t}^{AA} = \frac{3(\sqrt{S\psi}-1)^2}{\psi} - S$, so $\Pi_{F_t}^{UA*} > \Pi_{F_t}^{AA*}$ always holds.
- (ii-c) If $S > \frac{3(2-\sqrt{3})}{2\psi}$, the range for $\Pi_{F_t}^{UA*} \geq \Pi_{F_t}^{AA*}$ to hold is $S > \frac{3(2-\sqrt{3})}{2\psi}$ because $\Pi_{F_t}^{AA*} = 0$.

In summary, given $S > \tilde{\pi}_{V_n}^{AA}$, $\Pi_{F_t}^{AA*} \geq \Pi_{F_t}^{UA*}$ When $\tilde{\pi}_{V_n}^{AA} < S \leq max(S_{F_t}^{A2}, \tilde{\pi}_{V_n}^{AA})$; $\Pi_{F_t}^{AA*} < \Pi_{F_t}^{UA*}$ when $S > max(S_{F_t}^{A2}, \tilde{\pi}_{V_n}^{AA})$. □

[9] We could show $\tilde{\pi}_{V_n}^{AA} < \tilde{\pi}_{V_n}^{UA}$.

[10] It is easy to see $\frac{3(2-\sqrt{3})}{2\psi} < \frac{1}{\psi}$. $\frac{(9+2\phi r+2\psi c)^2}{144\psi} > \frac{9^2}{144\psi} > \frac{3(2-\sqrt{3})}{2\psi}$.

Table B.1: Payoff Matrix for the Farms' Adulteration Strategies when $0 < c \leq \frac{9(2-\sqrt{3})}{2\psi}$

$(\Pi_{F_t}^{x_t x_n *}, \Pi_{F_n}^{x_t x_n *})$		Non-traceable farm's strategy: A	Non-traceable farm's strategy: U
Traceable farm's strategy	A	$(\frac{(9-2\psi c)^2}{108\psi} - S, \frac{(9+2\psi c)^2}{108\psi})$ if $S \leq \tilde{\pi}_{V_n}^{AA}$ $(\frac{3(\sqrt{S\psi}-1)^2}{\psi} - S, \frac{\sqrt{S}(9+2\psi c)-12S\sqrt{\psi}}{2\sqrt{\psi}})$ if $\tilde{\pi}_{V_n}^{AA} < S \leq \frac{3(2-\sqrt{3})}{2\psi}$ (0,0) if $S > \frac{3(2-\sqrt{3})}{2\psi}$	$(\frac{(9+2\phi(\Delta q+r)-2\psi c)^2}{108\psi} - S, \frac{(9-2\phi(\Delta q+r)+2\psi c)^2}{108\psi})$
	U	$(\frac{(9-2\phi r-2\psi c)^2}{108\psi}, \frac{(9+2\phi r+2\psi c)^2}{108\psi})$ if $S \leq \tilde{\pi}_{V_n}^{UA}$ $(\frac{3(\sqrt{S\psi}-1)^2}{\psi}, \frac{\sqrt{S}(9+2\phi r+2\psi c)-12S\sqrt{\psi}}{2\sqrt{\psi}})$ if $\tilde{\pi}_{V_n}^{UA} < S \leq min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi})$ (0,0) if $S > min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi})$	$(\frac{(9+2\phi\Delta q-2\psi c)^2}{108\psi}, \frac{(9-2\phi\Delta q+2\psi c)^2}{108\psi})$

B.6. Proof of Theorem 1.

In our setting, a Nash equilibrium (x_t^*, x_n^*) is two farm's strategies, the traceable farm's adulteration strategy, and the non-traceable farm's adulteration strategy, if no unilateral deviation in strategy by the farm is profitable for him, that is, for the traceable farm, $\Pi_{F_t}^{x_t^* x_n^*} \geq \Pi_{F_t}^{x_t x_n^*}$; for the non-traceable farm, $\Pi_{F_n}^{x_t^* x_n^*} \geq \Pi_{F_n}^{x_t^* x_n}$. To begin with, we characterize conditions under which the demand for each type of product is positive. Specifically, we find the minimum demand for traceable and non-traceable products and make sure it is larger than 0. For traceable products, demand in scenario (U,A), i.e., $D_t(U,A) = \frac{9-2\psi c-2\phi r}{18}$, is the minimum demand among all four scenarios. Similarly, non-traceable products' demand in scenario (A,U), i.e., $D_n(A,U) = \frac{9+2\psi c-2\phi(\Delta q+r)}{18}$, is the minimum demand among all four scenarios. Hence, by solving the following two scenarios: (1) $D_n(A,U) > D_t(U,A) \geq 0$, and (2) $D_t(U,A) > D_n(A,U) \geq 0$, we can get conditions for demand larger than 0: $0 < c \leq \frac{9}{2\psi}$, $\frac{\psi c}{\phi} < \Delta q \leq \frac{9+2\psi c}{2\phi}$, $0 < r \leq r_m$, where

$$r_m = \begin{cases} \frac{9+2\psi c-2\phi\Delta q}{2\phi} & \text{if } \phi\Delta q > 2c\psi, \\ \frac{9-2\psi c}{2\phi} & \text{otherwise.} \end{cases} \tag{B.10}$$

Therefore, in the following analysis, we assume the above conditions for c, r, Δq. Next, we consider the farm's equilibrium adulteration strategy when $0 < c \leq \frac{9(2-\sqrt{3})}{2\psi}$, and subsequently, we consider the farm's equilibrium adulteration strategy when $\frac{9(2-\sqrt{3})}{2\psi} < c \leq \frac{9}{2\psi}$. When $0 < c \leq \frac{9(2-\sqrt{3})}{2\psi}$, each farm's adulteration strategy and the corresponding payoff are shown in the Table B.1.

(i) Scenario (U,U) is the Nash Equilibrium: $\Pi_{F_n}^{UU*} \geq \Pi_{F_n}^{UA*}$ and $\Pi_{F_t}^{UU*} \geq \Pi_{F_t}^{AU*}$.

The condition for $\Pi_{F_t}^{UU*} \geq \Pi_{F_t}^{AU*}$ is $S \geq S_{F_t}^U$, as shown in Lemma 1. The condition for $\Pi_{F_n}^{UU*} \geq \Pi_{F_n}^{UA*}$ is $S \geq min(S_{F_n}^U, \frac{1}{\psi})$, as shown in Lemma 3. Therefore, the condition for Nash equilibrium (U,U) is

$$S \geq max(min(S_{F_n}^U, \frac{1}{\psi}), S_{F_t}^U). \tag{B.11}$$

(ii) Scenario (A,U) is the Nash equilibrium: $\Pi_{F_n}^{AU*} \geq \Pi_{F_n}^{AA*}$ and $\Pi_{F_t}^{AU*} \geq \Pi_{F_t}^{UU*}$.

First, the condition for $\Pi_{F_t}^{AU*} \geq \Pi_{F_t}^{UU*}$ is $0 < S \leq S_{F_t}^U$, as shown in Lemma 1. The condition for $\Pi_{F_n}^{AU*} \geq \Pi_{F_n}^{AA*}$ is $S \geq min(S_{F_n}^A, \frac{3(2-\sqrt{3})}{2\psi})$, as shown in the Part (i) of Lemma 5. Therefore, the condition for Nash equilibrium (A,U) is

$$min(S_{F_n}^A, \frac{3(2-\sqrt{3})}{2\psi}) \leq S \leq S_{F_t}^U \tag{B.12}$$

Equation (B.12) could be refined to $\frac{3(2-\sqrt{3})}{2\psi} \leq S \leq S_{F_t}^U$ based on Lemma C2.

(iii) Scenario (U,A) is the Nash equilibrium: $\Pi_{F_n}^{UA*} \geq \Pi_{F_n}^{UU*}$ and $\Pi_{F_t}^{UA*} \geq \Pi_{F_t}^{AA*}$.

First, the condition for $\Pi_{F_n}^{UA*} \geq \Pi_{F_n}^{UU*}$ is $0 < S \leq min(S_{F_n}^{U}, \frac{1}{\psi})$, as shown in Lemma 3. The condition for $\Pi_{F_t}^{UA*} \geq \Pi_{F_t}^{AA*}$ is $min(S_{F_t}^{A1}, \tilde{\pi}_{V_n}^{AA}) \leq S \leq \tilde{\pi}_{V_n}^{AA}$ and $S \geq max(\tilde{\pi}_{V_n}^{AA}, S_{F_t}^{A2})$, as shown in Part (ii) of Lemma 5. Moreover, based on the proof of Lemma 3, we have $min(S_{F_n}^{U}, \frac{1}{\psi}) > \tilde{\pi}_{V_n}^{UA}$, and we can also show $\tilde{\pi}_{V_n}^{UA} > \tilde{\pi}_{V_n}^{AA}$ and $\tilde{\pi}_{V_n}^{UA} > S_{F_t}^{A2}$. Therefore, the condition for Nash equilibrium (U, A) is as follows,

$$S \in [min(S_{F_t}^{A1}, \tilde{\pi}_{V_n}^{AA}), \tilde{\pi}_{V_n}^{AA}] \cup [max(\tilde{\pi}_{V_n}^{AA}, S_{F_t}^{A2}), min(S_{F_n}^{U}, \tfrac{1}{\psi})] \tag{B.13}$$

Based on Lemma C1, the condition can be refined as $min(S_{F_t}^{A1}, S_{F_t}^{A2}) \leq S \leq min(S_{F_n}^{U}, \frac{1}{\psi})$.

(iv) Scenario (A,A) is the Nash equilibrium: $\Pi_{F_n}^{AA*} \geq \Pi_{F_n}^{AU*}$ and $\Pi_{F_t}^{AA*} \geq \Pi_{F_t}^{UA*}$.

First, based on Part (i) of Lemma 5, $\Pi_{F_n}^{AA*} \geq \Pi_{F_n}^{AU*}$ holds when $0 < S \leq \min(S_{F_n}^{A}, \frac{3(2-\sqrt{3})}{2\psi})$. The condition for $\Pi_{F_t}^{AA*} \geq \Pi_{F_t}^{UA*}$ is $0 < S \leq min(S_{F_t}^{A1}, \tilde{\pi}_{V_n}^{AA})$ and $\tilde{\pi}_{V_n}^{AA}) < S \leq max(\tilde{\pi}_{V_n}^{AA}, S_{F_t}^{A2})$.

Because $\tilde{\pi}_{V_n}^{AA} < S_{F_n}^{A}$ (shown in the Proof of Part (i) of Lemma 5) and $\tilde{\pi}_{V_n}^{AA} < \frac{3(2-\sqrt{3})}{2\psi}$, we have $min(S_{F_t}^{A1}, \tilde{\pi}_{V_n}^{AA}) < \min(S_{F_n}^{A}, \frac{3(2-\sqrt{3})}{2\psi})$. Therefore, the condition for Nash equilibrium (A, A) is as follows

$$S \in (0, min(S_{F_t}^{A1}, \tilde{\pi}_{V_n}^{AA})] \cup [\tilde{\pi}_{V_n}^{AA}, max(\tilde{\pi}_{V_n}^{AA}, S_{F_t}^{A2}) \cap \min(S_{F_n}^{A}, \tfrac{3(2-\sqrt{3})}{2\psi})]. \tag{B.14}$$

We can show that $S_{F_t}^{A2} < S_{F_n}^{A}$ by plugging $S_{F_n}^{A}$ into $\Lambda_{F_t}^{A2}(S)$ and getting $\Lambda_{F_t}^{A2}(S_{F_n}^{A}) < 0$. Additionally, in the proof of Lemma 5, $S_{F_t}^{A2} < \frac{3(2-\sqrt{3})}{2\psi}$. Based on these results and Lemma C1, Equation (B.14) can be refined to $0 < S \leq min(S_{F_t}^{A1}, S_{F_t}^{A2})$.

In conclusion, farms' adulteration strategies in equilibrium with respect to the government penalty when $0 < c \leq \frac{9(2-\sqrt{3})}{2\psi}$ are listed as follows:

$$(x_t^*, x_n^*) = \begin{cases} (U,U) \ if \ S > max(min(S_{F_n}^{U}, \frac{1}{\psi}), S_{F_t}^{U}), \\ (A,U) \ if \ \frac{3(2-\sqrt{3})}{2\psi} < S \leq S_{F_t}^{U}, \\ (U,A) \ if \ min(S_{F_t}^{A1}, S_{F_t}^{A2}) < S \leq min(S_{F_n}^{U}, \frac{1}{\psi}), \\ (A,A) \ if \ 0 < S \leq min(S_{F_t}^{A1}, S_{F_t}^{A2}). \end{cases}$$

Next, we consider the Nash equilibrium in Stage 1 when $\frac{9(2-\sqrt{3})}{2\psi} < c \leq \frac{9}{2\psi}$. The farm's adulteration strategies and the corresponding payoff are shown in Table B.2. Similarly, we can solve the conditions for different equilibria, and those conditions are listed as follows:

$$(x_t^*, x_n^*) = \begin{cases} (U,U) \ if \ S > max(min(S_{F_n}^{U}, \frac{1}{\psi}), S_{F_t}^{U}), \\ (A,U) \ if \ \tilde{\pi}_{F_t}^{AA} < S \leq S_{F_t}^{U}, \\ (U,A) \ if \ S_{F_t}^{A1} < S \leq min(S_{F_n}^{U}, \frac{1}{\psi}), \\ (A,A) \ if \ S \leq S_{F_t}^{A1}. \end{cases}$$

Additionally, we will find conditions that combine these two cases ($0 < c \leq \frac{9(2-\sqrt{3})}{2\psi}$ and $\frac{9(2-\sqrt{3})}{2\psi} < c \leq \frac{9}{2\psi}$). Define $\Gamma = \tilde{\pi}_{F_t}^{AA} - \frac{3(2-\sqrt{3})}{2\psi}$, and Γ is convex in c since $\frac{\mathrm{d}^2\Gamma}{\mathrm{d}c^2} = \frac{2\psi}{27} > 0$. Additionally, $\frac{\mathrm{d}\Gamma}{\mathrm{d}c} = 0$ at $c = \frac{9}{2\psi}$, so Γ decreases with c when $0 < c \leq \frac{9}{2\psi}$. Additionally, plug $\frac{9(2-\sqrt{3})}{2\psi}$ into Γ, we have $\Gamma(\frac{9(2-\sqrt{3})}{2\psi}) = 0$. Hence, when $\frac{9(2-\sqrt{3})}{2\psi} < c \leq \frac{9}{2\psi}$, $\tilde{\pi}_{F_t}^{AA} < \frac{3(2-\sqrt{3})}{2\psi}$; otherwise, $\tilde{\pi}_{F_t}^{AA} \geq \frac{3(2-\sqrt{3})}{2\psi}$ when $0 < c \leq \frac{9(2-\sqrt{3})}{2\psi}$ Furthermore, recall from Lemma C1, $S_{F_t}^{A2} > S_{F_t}^{A1}$ when $\frac{9(2-\sqrt{3})}{2\psi} < c \leq \frac{9}{2\psi}$. Therefore, the Nash equilibrium in Stage 1 can be rewritten as follows:

$$(x_t^*, x_n^*) = \begin{cases} (U,U) \ if \ S > max(min(S_{F_n}^{U}, \frac{1}{\psi}), S_{F_t}^{U}), \\ (A,U) \ if \ min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi}) < S \leq S_{F_t}^{U}, \\ (U,A) \ if \ min(S_{F_t}^{A1}, S_{F_t}^{A2}) < S \leq min(S_{F_n}^{U}, \frac{1}{\psi}), \\ (A,A) \ if \ 0 < S \leq min(S_{F_t}^{A1}, S_{F_t}^{A2}). \end{cases}$$

Table B.2: Payoff Matrix for the Farms' Adulteration Strategies when $\frac{9(2-\sqrt{3})}{2\psi} < c \leq \frac{9}{2\psi}$

$(\Pi_{F_t}^{x_t x_n *}, \Pi_{F_n}^{x_t x_n *})$		Non-traceable farm's strategy	
		A	U
Traceable farm's strategy	A	$(\frac{(9-2\psi c)^2}{108\psi} - S, \frac{(9+2\psi c)^2}{108\psi})$ if $S \leq \tilde{\pi}_{F_t}^{AA}$ (0,0) if $S > \tilde{\pi}_{F_t}^{AA}$	$(\frac{(9+2\phi(\Delta q+r)-2\psi c)^2}{108\psi} - S, \frac{(9-2\phi(\Delta q+r)+2\psi c)^2}{108\psi})$
	U	$(\frac{(9-2\phi r-2\psi c)^2}{108\psi}, \frac{(9+2\phi r+2\psi c)^2}{108\psi})$ if $S \leq \tilde{\pi}_{V_n}^{UA}$ $(\frac{3(\sqrt{S\psi}-1)^2}{\psi}, \frac{\sqrt{S}(9+2\phi r+2\psi c)-12S\sqrt{\psi}}{2\sqrt{\psi}})$ if $\tilde{\pi}_{V_n}^{UA} < S \leq min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi})$ (0,0) if $S > min(\frac{1}{\psi}, \frac{(9+2\phi r+2\psi c)^2}{144\psi})$	$(\frac{(9+2\phi\Delta q-2\psi c)^2}{108\psi}, \frac{(9-2\phi\Delta q+2\psi c)^2}{108\psi})$

Since $S_{F_t}^{A1} > 0$ and $S_{F_t}^{A2} > 0$, equilibrium (A,A) always exists when $0 < S \leq min(S_{F_t}^{A1}, S_{F_t}^{A2})$. Similarly, equilibrium (U,U) always exists when $S > max(min(S_{F_n}^{U}, \frac{1}{\psi}), S_{F_t}^{U})$. Additionally, based on the proof of Lemma 3 and Lemma 5, we have $S_{F_n}^{U} > \tilde{\pi}_{V_n}^{UA} > S_{F_t}^{A2}$ and $\frac{1}{\psi} > \frac{3(2-\sqrt{3})}{2\psi} > S_{F_t}^{A2}$; hence, $min(S_{F_n}^{U}, \frac{1}{\psi}) > min(S_{F_t}^{A1}, S_{F_t}^{A2})$, and equilibrium (U,A) always exists when $min(S_{F_t}^{A1}, S_{F_t}^{A2}) < S \leq min(S_{F_n}^{U}, \frac{1}{\psi})$. While for the equilibrium only traceable farm adulterates (A,U), the region might be vacant because $min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi})$ might be greater than $S_{F_t}^{U}$ under certain market conditions. Additionally, we know that $\frac{3(2-\sqrt{3})}{2\psi} > S_{F_t}^{A2}$, and $\tilde{\pi}_{F_t}^{AA} > S_{F_t}^{A1} > S_{F_t}^{A2}$ if $\tilde{\pi}_{F_t}^{AA} < \frac{3(2-\sqrt{3})}{2\psi}$. Hence, $min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi}) > min(S_{F_t}^{A1}, S_{F_t}^{A2})$. Therefore, when

$$min(\tilde{\pi}_{F_t}^{AA}, \frac{3(2-\sqrt{3})}{2\psi}) < S < min(S_{F_n}^{U}, \frac{1}{\psi}, S_{F_t}^{U}), \tag{B.15}$$

both scenarios (A,U) and (U,A) are the equilibria. □

B.7. Proof of Proposition 1.

(i) **Existence of Region $\mathbb{P}$, where**

$$\mathbb{P} = \{S : min(S_{F_n}^{U}, \frac{1}{\psi}) < S < S_{F_t}^{U}\}.$$

From Lemma C3, we know that $S_{F_n}^{U} > \frac{1}{\psi}$ if $S_{F_t}^{U} > \frac{1}{\psi}$. Hence, in the following, we consider the conditions where Region $\mathbb{P}$ exists, that is, $S_{F_t}^{U} > \frac{1}{\psi}$. Recall from Lemma 1,

$$S_{F_t}^{U} = \frac{\phi r(9+2\phi\Delta q+\phi r-2\psi c)}{27\psi}.$$

$S_{F_t}^{U}$ is convex increasing in r when $r > 0$. When $r = 0$, $S_{F_t}^{U} = 0$. We also have

$$\lim_{r \to +\infty} S_{F_t}^{U} \to +\infty.$$

On the other hand, $\frac{1}{\psi}$ is independent of r. Therefore, there must exist a unique

$$r_{au} = \frac{-9-2\Delta q\phi+2c\psi}{2\phi} + \frac{1}{2}\sqrt{\frac{189+36\Delta q\phi+4\Delta q^2\phi^2-36c\psi-8\Delta qc\phi\psi+4c^2\psi^2}{\phi^2}},$$

such that $S_{F_t}^{U}(r_{au}) = \frac{1}{\psi}$. Hence, $S_{F_t}^{U} > \frac{1}{\psi}$ when $r > r_{au}$.

Next, we compare r_{au} with r_m (defined in Equation (B.10)). If $r_{au} > r_m$, then $S_{F_t}^{U} < \frac{1}{\psi}$ when $0 < r \leq r_m$; otherwise, if $r_{au} \leq r_m$, $S_{F_t}^{U} > \frac{1}{\psi}$ when $r_{au} < r \leq r_m$. As shown in Lemma B1, $r_{au} \leq r_m$ when

$$0 < c \leq \frac{6\sqrt{6}-9}{2\psi} \text{ and } \max\left(\frac{\psi c}{\phi}, \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}\right) < \Delta q \leq \frac{6\sqrt{6}-9+2\psi c}{2\phi}.$$

(ii) **Existence of Region $\mathbb{R}_I$, where**

$$\mathbb{R}_I = \{S : min(\tilde{\pi}^{AA}_{F_t}, \frac{3(2-\sqrt{3})}{2\psi}) < S \leq min(\tilde{S}, S^U_{F_t}, \tilde{\pi}^{UA}_{V_n})\}.$$

We relax the constraint by considering $min(\tilde{\pi}^{AA}_{F_t}, \frac{3(2-\sqrt{3})}{2\psi}) < S \leq min(\tilde{S}, S^U_{F_t})$. Then we consider the following two cases.

(ii-a) If $S^U_{F_t} < min(\tilde{\pi}^{AA}_{F_t}, \frac{3(2-\sqrt{3})}{2\psi})$, $\mathbb{R}_I$ does not exist.

(ii-b) If $S^U_{F_t} > min(\tilde{\pi}^{AA}_{F_t}, \frac{3(2-\sqrt{3})}{2\psi})$, we then compare $\tilde{S}$ with $min(\tilde{\pi}^{AA}_{F_t}, \frac{3(2-\sqrt{3})}{2\psi})$. From Equation (A.3), we can get

$$\tilde{S} = \frac{\phi\Big(2c\psi\Big(-18\phi\Big(\Delta q^2-2\Delta qr-2r^2\Big)-8r\phi^2(\Delta q+r)(\Delta q+2r)+81(\Delta q+2r)\Big)-8c^3\psi^3(\Delta q+2r)+4\Delta qc^2\psi^2(\Delta q\phi+2r\phi+9)+\Delta q(\Delta q\phi+2r\phi-9)\Big(8r\phi^2(\Delta q+r)+81\Big)\Big)}{27\psi(-2\phi(\Delta q+r)+2c\psi+9)^2}$$

Taking the first-order derivative with respect to r, we have

$$\frac{\mathrm{d}\tilde{S}}{\mathrm{d}r} = -\frac{2M\phi(\Delta q\phi-2c\psi)}{27\psi(-2\phi(\Delta q+r)+2c\psi+9)^3},$$

where

$$M = 8\Delta q^3\phi^3 + 4\Delta q^2\phi^2(-2c\psi+10r\phi-27) + 12\Delta q\phi(2r\phi-3)(-2c\psi+2r\phi-9)$$
$$-24r^2\phi^2(2c\psi+9) - 2r\phi(2c\psi-27)(2c\psi+9) - (2c\psi-9)(2c\psi+9)^2 + 16r^3\phi^3 > 0.$$

So we have $\frac{\mathrm{d}\tilde{S}}{\mathrm{d}r} < 0$ when $\Delta q > \frac{2\psi c}{\phi}$, and $\frac{\mathrm{d}\tilde{S}}{\mathrm{d}r} > 0$ when $\Delta q \leq \frac{2\psi c}{\phi}$. Next, plug $r = 0$ into $\tilde{S}$, we have

$$\tilde{S} = \frac{\Delta q\phi(9-2c\psi)^2(\Delta q\phi-2c\psi-9)}{27\psi(-2\Delta q\phi+2c\psi+9)^2} < 0.$$

Hence, we consider the following two subcases:

(1) When $\Delta q > \frac{2\psi c}{\phi}$, $\tilde{S} < 0$ and Region $\mathbb{R}_I$ does not exist.

(2) When $\Delta q \leq \frac{2\psi c}{\phi}$, plug $r = r_m$ (See Equation (B.10), $r_m = \frac{9-2\psi c}{2\phi}$ if $\Delta q \leq \frac{2\psi c}{\phi}$) into $\tilde{S}$, we have

$$\tilde{S} = \frac{(9-2c\psi)(4\Delta q\phi-6c\psi+27)}{108\psi} > min(\tilde{\pi}^{AA}_{F_t}, \frac{3(2-\sqrt{3})}{2\psi}).$$

Hence, Region $\mathbb{R}_I$ always exists when $min(\tilde{\pi}^{AA}_{F_t}, \frac{3(2-\sqrt{3})}{2\psi}) < S^U_{F_t}$ and $\Delta q \leq \frac{2\psi c}{\phi}$. Combining the condition where $S^U_{F_t} > min(\tilde{\pi}^{AA}_{F_t}, \frac{3(2-\sqrt{3})}{2\psi})$ (See Lemma B2) and $\Delta q \leq \frac{2\psi c}{\phi}$, we have Region $\mathbb{R}_I$ exists when $0 < c < \frac{9}{2\psi}$ and $\frac{\psi c}{\phi} < \Delta q \leq \frac{2\psi c}{\phi}$.

(iii) **Existence of Region $\mathbb{R}_{II}$, where**

$$\mathbb{R}_{II} = \{S : \mathcal{RD} \cap min(S^U_{F_t}, \tilde{\pi}^{UA}_{V_n}) < S \leq min(S^U_{F_t}, \frac{1}{\psi})\}.$$

In Section A.3, we show that $\mathcal{RD}$ is the region consisting of S_0 and its interior, where S_0 is the solution to $\bar{R}(S) = 0$. If part of S_0 satisfies $min(S^U_{F_t}, \tilde{\pi}^{UA}_{V_n}) < S_0 \leq min(S^U_{F_t}, \frac{1}{\psi})$, then Region $\mathbb{R}_{II}$ exists. Next, we first characterize conditions under which $min(S^U_{F_t}, \tilde{\pi}^{UA}_{V_n}) < min(S^U_{F_t}, \frac{1}{\psi})$, and then figure out conditions for $min(S^U_{F_t}, \tilde{\pi}^{UA}_{V_n}) < S_0 \leq min(S^U_{F_t}, \frac{1}{\psi})$.

It is easy to show $\frac{1}{\psi} > \tilde{\pi}^{UA}_{V_n}$. Hence, $min(S^U_{F_t}, \tilde{\pi}^{UA}_{V_n}) < S \leq min(S^U_{F_t}, \frac{1}{\psi})$ is feasible if and only if $S^U_{F_t} > \tilde{\pi}^{UA}_{V_n}$. As shown in Lemma C4, $S^U_{F_t} \geq \tilde{\pi}^{UA}_{V_n}$ when

$$0 < c \leq \frac{6\sqrt{6}-9}{2\psi}, \max\left(\frac{\psi c}{\phi}, \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}\right) < \Delta q \leq \frac{10c\psi+\sqrt{3}\sqrt{(9-2c\psi)(2c\psi+63)}-9}{8\phi} \text{ and } r_d \leq r \leq r_m.$$

Combining the conditions for $S^U_{F_t} > \frac{1}{\psi}$ (shown in Lemma B1), we consider the following two cases.

Table B.3: Conditions for the Existence of Different Regions

Existence of Regions	c	Δq
(a) With region $\mathbb{R}_I, \mathbb{R}_{II}$ and $\mathbb{P}$	$0 < c \leq \frac{6\sqrt{6}-9}{2\psi}$	$\max\left(\frac{\psi c}{\phi}, \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}\right) < \Delta q \leq \frac{2\psi c}{\phi}$
(b) No region $\mathbb{R}_I$	$0 < c \leq \frac{6\sqrt{6}-9}{2\psi}$	$\frac{2\psi c}{\phi} < \Delta q \leq \frac{6\sqrt{6}-9+2c\psi}{2\phi}$
(c) No region $\mathbb{R}_{II}$ or $\mathbb{P}$	$\frac{18-3\sqrt{2}}{2\psi} < c \leq \frac{6\sqrt{6}-9}{2\psi}$	$\frac{\psi c}{\phi} < \Delta q \leq \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}$
(d) No region $\mathbb{R}_{II}$ or $\mathbb{P}$	$\frac{6\sqrt{6}-9}{2\psi} < c \leq \frac{9}{2\psi}$	$\frac{\psi c}{\phi} < \Delta q \leq \frac{2\psi c}{\phi}$
(e) No region $\mathbb{R}_I, \mathbb{R}_{II}$ or $\mathbb{P}$	$\frac{6\sqrt{6}-9}{2\psi} < c \leq \frac{9}{2\psi}$	$\frac{2\psi c}{\phi} < \Delta q \leq \frac{6\sqrt{6}-9+2c\psi}{2\phi}$
(f) No region $\mathbb{R}_I, \mathbb{R}_{II}$ or $\mathbb{P}$	$0 < c \leq \frac{9}{2\psi}$	$\frac{6\sqrt{6}-9+2c\psi}{2\phi} < \Delta q \leq \frac{9+2c\psi}{2\phi}$

(iii-a) $0 < c \leq \frac{6\sqrt{6}-9}{2\psi}$ and $\max\left(\frac{\psi c}{\phi}, \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}\right) < \Delta q \leq \frac{6\sqrt{6}-9+2\psi c}{2\phi}$.[11] In this case, $S_{F_t}^U > \frac{1}{\psi}$ when $r_{au} < r \leq r_m$. Since $\frac{1}{\psi} > \tilde{\pi}_{V_n}^{UA}$, we have $r_d < r_{au} \leq r_m$. Then we consider the range in which $r_{au} < r \leq r_m$.

If $r_{au} < r \leq r_m$, $\tilde{\pi}_{V_n}^{UA} < \frac{1}{\psi} < S_{F_t}^U$. When $S = \frac{1}{\psi}$, we can show that $\bar{R}(\frac{1}{\psi}) > 0$ ($\bar{R}$ is defined in Equation (A.5)) since $\bar{R}^{UA}(\frac{1}{\psi}) = 0$ and $\bar{R}^{AU}(\frac{1}{\psi}) > 0$. Additionally, when $S = \tilde{\pi}_{V_n}^{UA}$, we can show that $\bar{R}(\tilde{\pi}_{V_n}^{UA}) = \tilde{R}(\tilde{\pi}_{V_n}^{UA}) < 0$ if $\tilde{\pi}_{V_n}^{UA} > \tilde{S}$[12]. Hence, there exists a part of S_0 satisfying $\bar{R}(S_0) = 0$ when $\tilde{\pi}_{V_n}^{UA} < S < \frac{1}{\psi}$.

(iii-b) $0 < c \leq \frac{6\sqrt{6}-9}{2\psi}$ and $\frac{6\sqrt{6}-9+2\psi c}{2\phi} < \Delta q \leq \frac{10c\psi+\sqrt{3}\sqrt{(9-2c\psi)(2c\psi+63)}-9}{8\phi}$. In this case, $\tilde{\pi}_{V_n}^{UA} < S_{F_t}^U < \frac{1}{\psi}$ if $r_d < r \leq r_m$. When $S = S_{F_t}^U$, we can show that $\bar{R}(S_{F_t}^U) < 0$ since $\bar{R}^{UA}(S_{F_t}^U) > 0$ and $\bar{R}^{AU}(S_{F_t}^U) = 0$. Additionally, when $S = \tilde{\pi}_{V_n}^{UA}$, we can show that $\bar{R}(\tilde{\pi}_{V_n}^{UA}) = \tilde{R}(\tilde{\pi}_{V_n}^{UA}) < 0$. Therefore, when $\tilde{\pi}_{V_n}^{UA} < S < S_{F_t}^U$, $\bar{R}(S) < 0$ and (U,A) dominates (A,U).

Combining case (iii-a) and (iii-b), we find that Region $\mathbb{R}_{II}$ exists when Region $\mathbb{P}$ exists, that is,

$$0 < c \leq \frac{6\sqrt{6}-9}{2\psi} \text{ and } \max\left(\frac{\psi c}{\phi}, \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}\right) < \Delta q \leq \frac{6\sqrt{6}-9+2\psi c}{2\phi}.$$

In summary, Table B.3 illustrates the existence of region $\mathbb{R}_{II}$, $\mathbb{R}_{II}$, and $\mathbb{P}$. □

LEMMA B1. *$r_{au} \leq r_m$ when $0 < c \leq \frac{6\sqrt{6}-9}{2\psi}$ and $\max\left(\frac{\psi c}{\phi}, \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}\right) < \Delta q \leq \frac{6\sqrt{6}-9+2\psi c}{2\phi}$.*

B.8. Proof of Lemma B1.

In Proposition 1, we have shown that $S_{F_t}^U$ is convex increasing in r when $r > 0$ and $S_{F_t}^U = 0$ when $r = 0$. Additionally, $\frac{1}{\psi}$ is independent of r. Hence, when $r > r_{au}$, $S_{F_t}^U > \frac{1}{\psi}$. Define $W(r, \Delta q) = S_{F_t}^U - \frac{1}{\psi}$ and r_{au} satisfies $W(r_{au}, \Delta q) = 0$. Next, in the following analysis, we plug r_m into W, and if $W(r_m, \Delta q) \geq 0$, $r_m \geq r_{au}$. We consider the following two cases with respect to different values of r_m.

(i) If $\frac{\psi c}{\phi} < \Delta q \leq \frac{2\psi c}{\phi}$, $r_m = \frac{9-2c\psi}{2\phi}$. Plug r_m into W, we have

$$W(r_m, \Delta q) = \frac{4\Delta q\phi(9-2c\psi)+3(2c\psi-15)(2c\psi-3)}{108\psi}.$$

$W(r_m, \Delta q)$ increases with Δq since $\frac{\partial W(r_m,\Delta q)}{\partial \Delta q} = \frac{\phi(9-2c\psi)}{27\psi} > 0$. Therefore, there must exist a

$$\Delta\hat{q} = \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}$$

such that $W(r_m, \Delta\hat{q}) = 0$. Additionally, we can get $\Delta\hat{q} \leq \frac{2\psi c}{\phi}$ when $0 < c \leq \frac{6\sqrt{6}-9}{2\psi}$. Therefore, when $0 < c \leq \frac{6\sqrt{6}-9}{2\psi}$ and $\max\left(\frac{\psi c}{\phi}, \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}\right) < \Delta q \leq \frac{2\psi c}{\phi}$, $W(r_m, \Delta q) \geq 0$ and $r_{au} \leq r_m$.

[11] $\frac{6\sqrt{6}-9+2\psi c}{2\phi} < \frac{10c\psi+\sqrt{3}\sqrt{(9-2c\psi)(2c\psi+63)}-9}{8\phi}$.

[12] It is necessary to show $\tilde{\pi}_{V_n}^{UA} > \tilde{S}$ when $\max\left(\frac{\psi c}{\phi}, \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}\right) < \Delta q \leq \frac{6\sqrt{6}-9+2\psi c}{2\phi}$

(ii) If $\frac{2\psi c}{\phi} < \Delta q \le \frac{9+2\psi c}{2\phi}$, $r_m = \frac{9+2c\psi-2\phi\Delta q}{2\phi}$. Plug r_m into W, we have

$$W(r_m, \Delta q) = \frac{-4\Delta q^2\phi^2+4\Delta q\phi(2c\psi-9)-4c\psi(c\psi-9)+135}{108\psi}.$$

$W(r_m, \Delta q)$ is concave in Δq since $\frac{\partial^2 W(r_m,\Delta q)}{\partial \Delta q^2} = -\frac{2\phi^2}{27\psi} < 0$. Therefore, there must exist a

$$\Delta\tilde{q} = \frac{2c\psi+6\sqrt{6}-9}{2\phi}$$

such that $W(r_m, \Delta\tilde{q}) = 0$. The other solution to $W(r_m, \Delta q) = 0$ is $\Delta\tilde{q}_1 = \frac{2c\psi-6\sqrt{6}-9}{2\phi} < 0$. Similarly, we can show that $\frac{2c\psi}{\phi} \le \Delta\tilde{q} < \frac{9+2\psi c}{2\phi}$ when $0 < c \le \frac{6\sqrt{6}-9}{2\psi}$; otherwise, when $c > \frac{6\sqrt{6}-9}{2\psi}$, $\Delta\tilde{q} < \frac{2c\psi}{\phi}$. In conclusion, when $0 < c \le \frac{6\sqrt{6}-9}{2\psi}$ and $\frac{2\psi c}{\phi} < \Delta q \le \frac{6\sqrt{6}-9+2\psi c}{2\phi}$, $W(r_m, \Delta q) \ge 0$, and we have $r_{au} \le r_m$. Combining Case (i) and (ii), we have $r_{au} \le r_m$ when $0 < c \le \frac{6\sqrt{6}-9}{2\psi}$ and $\max\left(\frac{\psi c}{\phi}, \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}\right) < \Delta q \le \frac{6\sqrt{6}-9+2\psi c}{2\phi}$. □

Lemma B2. $S^U_{F_t} > min(\tilde{\pi}^{AA}_{F_t}, \frac{3(2-\sqrt{3})}{2\psi})$ *when*

(i) $0 < c \le \frac{9(2-\sqrt{3})}{2\psi}$, $\frac{\psi c}{\phi} < \Delta q < \frac{9(3^{1/4}\sqrt{2}-1)+2c\psi}{2\phi}$, *and* $r_1 < r \le r_m$; r_1 *is defined in Equation (B.16).*

(ii) $\frac{9(2-\sqrt{3})}{2\psi} < c \le \frac{9}{2\psi}$, $\frac{\psi c}{\phi} < \Delta q < \frac{-9+2\psi c+\sqrt{(27-2\psi c)(9+2\psi c)}}{2\phi}$ *and* $r_2 < r \le r_m$; r_2 *is defined in Equation (B.17).*

B.9. Proof of Lemma B2.

In Proposition 1, we have shown that $S^U_{F_t}$ is convex increasing in r when $r > 0$ and $S^U_{F_t} = 0$ when $r = 0$. Additionally, $\tilde{\pi}^{AA}_{F_t}$ and $\frac{3(2-\sqrt{3})}{2\psi}$ are independent of r. Therefore, there must exist a unique

$$r_1 = \frac{-9-2\Delta q\phi+2c\psi}{2\phi} + \frac{1}{2}\sqrt{\frac{405-162\sqrt{3}+36\Delta q\phi+4\Delta q^2\phi^2-36c\psi-8\Delta qc\phi\psi+4c^2\psi^2}{\phi^2}} \tag{B.16}$$

such that $S^U_{F_t}(r_1) = \frac{3(2-\sqrt{3})}{2\psi}$. Similarly, there must exist a unique

$$r_2 = \frac{-9-2\phi\Delta q+2\psi c+\sqrt{2(2\phi^2\Delta q^2+2\phi\Delta q(9-2\psi c)+(9-2\psi c)^2)}}{2\phi}, \tag{B.17}$$

such that $S^U_{F_t}(r_2) = \tilde{\pi}^{AA}_{F_t}$. Hence, we have $S^U_{F_t} > min(\tilde{\pi}^{AA}_{F_t}, \frac{3(2-\sqrt{3})}{2\psi})$ when $r > min(r_1, r_2)$.

In Theorem 1, we have shown that $\tilde{\pi}^{AA}_{F_t} < \frac{3(2-\sqrt{3})}{2\psi}$ $(r_2 < r_1)$ when $\frac{9(2-\sqrt{3})}{2\psi} < c \le \frac{9}{2\psi}$; otherwise, $\tilde{\pi}^{AA}_{F_t} \ge \frac{3(2-\sqrt{3})}{2\psi}$ $(r_2 \ge r_1)$ when $0 < c \le \frac{9(2-\sqrt{3})}{2\psi}$. Hence, we consider the following two scenarios.

(i) When $0 < c \le \frac{9(2-\sqrt{3})}{2\psi}$, $r_1 \le r_2$. Hence, we need to show conditions under which $r_1 \le r_m$. Define $O(r, \Delta q) = S^U_{F_t} - \frac{3(2-\sqrt{3})}{2\psi}$ and r_1 satisfies $O(r_1, \Delta q) = 0$. Next, in the following analysis, we plug r_m into O, and if $O(r_m, \Delta q) \ge 0$, $r_m \ge r_1$. We consider the following two subcases with respect to different values of r_m.

(i-a) When $\frac{\psi c}{\phi} < \Delta q \le \frac{2\psi c}{\phi}$, $r_m = \frac{9-2\psi c}{2\phi}$. Plug r_m into O, we have

$$O(r_m, \Delta q) = \frac{(9-2\psi c)(27+4\phi\Delta q-6\psi c)}{108\psi} - \frac{3(2-\sqrt{3})}{2\psi}.$$

$O(r_m, \Delta q)$ increases with Δq since $\frac{\partial O(r_m,\Delta q)}{\partial \Delta q} = \frac{\phi(9-2\psi c)}{27\psi} > 0$. Define

$$\Delta q_1 = \frac{-81+162\sqrt{3}-108\psi c+12\psi^2c^2}{-36\phi+8c\psi\phi}$$

as the solution to $O(r_m, \Delta q) = 0$. We can show that $\Delta q_1 < 0$ given $0 < c \le \frac{9(2-\sqrt{3})}{2\psi}$. Therefore, $r_1 < r_m$ when $0 < c \le \frac{9(2-\sqrt{3})}{2\psi}$ and $\frac{\psi c}{\phi} < \Delta q \le \frac{2\psi c}{\phi}$.

(i-b) When $\frac{2\psi c}{\phi} < \Delta q \le \frac{9+2\psi c}{2\phi}$, $r_m = \frac{9-2\phi\Delta q+2\psi c}{2\phi}$. Plug r_m into O, we have

$$O(r_m, \Delta q) = \frac{(27+2\phi\Delta q-2\psi c)(9-2\phi\Delta q+2\psi c)}{108\psi} - \frac{3(2-\sqrt{3})}{2\psi}.$$

$O(r_m, \Delta q)$ is concave in Δq. Solve for $O(r_m, \Delta q) = 0$, we can get

$$\Delta q_2 = \frac{9(3^{1/4}\sqrt{2}-1)+2c\psi}{2\phi} > 0,$$

$\Delta q_3 = \frac{-9(3^{1/4}\sqrt{2}+1)+2c\psi}{2\phi} < 0$. Then we can show that $\frac{2\psi c}{\phi} < \Delta q_2 < \frac{9+2\psi c}{2\phi}$ given $0 < c \leq \frac{9(2-\sqrt{3})}{2\psi}$. Thus, $r_1 \leq r_m$ when $0 < c \leq \frac{9(2-\sqrt{3})}{2\psi}$ and $\frac{2\psi c}{\phi} < \Delta q \leq \Delta q_2$. Combining Case (i-a) and (i-b), we get results in Lemma B2-(i).

(ii) When $\frac{9(2-\sqrt{3})}{2\psi} < c \leq \frac{9}{2\psi}$, $r_2 < r_1$. We need to show conditions under which $r_2 \leq r_m$. Define $\hat{O}(r, \Delta q) = S^U_{F_t} - \tilde{\pi}^{AA}_{F_t}$ and r_2 satisfies $\hat{O}(r_2, \Delta q) = 0$. Next, in the following analysis, we plug r_m into $\hat{O}$, and if $\hat{O}(r_m, \Delta q) \geq 0$, $r_m \geq r_2$. We consider the following two subcases with respect to different values of r_m.

(ii-a) When $\frac{\psi c}{\phi} < \Delta q \leq \frac{2\psi c}{\phi}$, $r_m = \frac{9-2\psi c}{2\phi}$. Plug r_m into $\hat{O}$, we have

$$\hat{O}(r_m, \Delta q) = \frac{(9-2\psi c)(27+4\Delta q\phi-6\psi c)}{108\psi}.$$

$\hat{O}(r_m, \Delta q)$ increases with Δq since $\frac{\partial \hat{O}(r,\Delta q)}{\partial \Delta q} = \frac{\phi(9-2\psi c)}{27\psi} > 0$. Define

$$\Delta q_4 = \frac{-9+2\psi c}{2\phi} < 0$$

as the solution to $\hat{O}(r_m, \Delta q) = 0$. Therefore, $r_2 < r_m$ given $\frac{\psi c}{\phi} < \Delta q \leq \frac{2\psi c}{\phi}$.

(ii-b) When $\frac{2\psi c}{\phi} < \Delta q \leq \frac{9+2\psi c}{2\phi}$, $r_m = \frac{9-2\phi\Delta q+2\psi c}{2\phi}$. Plug r_m into $\hat{O}$, we have

$$\hat{O}(r_m, \Delta q) = \frac{81-2\Delta q^2\phi^2+4c\psi(9-c\psi)-2\Delta q\phi(9-2c\psi)}{54\psi}.$$

$\hat{O}(r_m, \Delta q)$ is concave in Δq since $\frac{\partial^2 \hat{O}(\Delta q)}{\partial \Delta q^2} = -\frac{2\phi^2}{27\psi} < 0$. Solve for $\hat{O}(r_m, \Delta q) = 0$, we can get

$$\Delta q_5 = \frac{-9+2\psi c+\sqrt{(27-2\psi c)(9+2\psi c)}}{2\phi},$$

$\Delta q_6 = \frac{-9+2\psi c-\sqrt{(27-2\psi c)(9+2\psi c)}}{2\phi} < 0$. Additionally, we can show that $\frac{2\psi c}{\phi} < \Delta q_5 \leq \frac{9+2\psi c}{2\phi}$. Thus, $r_2 \leq r_m$ when $\frac{2\psi c}{\phi} < \Delta q \leq \Delta q_5$. Combining Case (ii-a) and (ii-b), we get results in Lemma B2-(ii). □

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Appendix C: Additional Analytical Results

LEMMA C1. *(a) When* $0 < c \le \frac{9(2-\sqrt{3})}{2\psi}$*, there exists a* $r^A_{F_t}$ *such that when* $0 < r \le r^A_{F_t}$, $S^{A1}_{F_t} \le S^{A2}_{F_t} \le \tilde{\pi}^{AA}_{V_n}$*; otherwise, when* $r^A_{F_t} < r \le r_m$, $S^{A1}_{F_t} > S^{A2}_{F_t} > \tilde{\pi}^{AA}_{V_n}$*. (b) When* $\frac{9(2-\sqrt{3})}{2\psi} < c \le \frac{9}{2\psi}$, $\tilde{\pi}^{AA}_{V_n} > S^{A2}_{F_t} > S^{A1}_{F_t}$.

C.1. Proof of Lemma C1.

Define

$$g(r) = S^{A1}_{F_t} - \tilde{\pi}^{AA}_{V_n} = \frac{\phi r(9-\phi r-2\psi c)}{27\psi} - \frac{(9+2\psi c)^2}{324\psi} = -\frac{12r^2\phi^2+12r\phi(-9+2c\psi)+(9+2c\psi)^2}{324\psi}.$$

It is obvious to show $g(r)$ is concave in r so that $g(r)$ has a maximum value, which is defined as $g_m = \frac{81+4\psi c(-18+c\psi)}{162\psi}$. $g_m < 0$ when $\frac{9(2-\sqrt{3})}{2\psi} < c \le \frac{9}{2\psi}$; otherwise, $g_m \ge 0$ when $0 < c \le \frac{9(2-\sqrt{3})}{2\psi}$. If $g_m \ge 0$, there must exist two thresholds $r^A_{F_t} = \frac{27-6c\psi-\sqrt{6(81+4c\psi(-18+c\psi))}}{6\phi}$ and $\hat{r}^A_{F_t} = \frac{27-6c\psi+\sqrt{6(81+4c\psi(-18+c\psi))}}{6\phi}$ such that $g(r^A_{F_t}) = 0$ and $g(\hat{r}^A_{F_t}) = 0$. Comparing $r^A_{F_t}$, $\hat{r}^A_{F_t}$ with r_m respectively. It is easy to show $0 < r^A_{F_t} \le r_m$ (See Equation (B.10)) and $\hat{r}^A_{F_t} > r_m$. In summary, $g(r) \ge 0$ if $0 < c \le \frac{9(2-\sqrt{3})}{2\psi}$ and $r^A_{F_t} \le r \le r_m$; otherwise, $g(r) < 0$.

Plug $\tilde{\pi}^{AA}_{V_n}$ into $\Lambda^{A2}_{F_t}(S)$, we can get

$$\Lambda^{A2}_{F_t}(\tilde{\pi}^{AA}_{V_n}) = -\frac{12r^2\phi^2+12r\phi(-9+2c\psi)+(9+2c\psi)^2}{324\psi} = g(r).$$

Plug $S^{A1}_{F_t}$ into $\Lambda^{A2}_{F_t}(S)$, we can get

$$\begin{aligned}\Lambda^{A2}_{F_t}(S^{A1}_{F_t}) &= -\frac{(9-2c\psi)^2-4(-9+\sqrt{3}\sqrt{-r\phi(-9+r\phi+2c\psi)})^2}{108\psi} \\ &= \frac{(2\sqrt{3(r\phi(-2c\psi-r\phi+9))}-(9-2c\psi)-18)(2\sqrt{3(r\phi(-2c\psi-r\phi+9))}+(9-2c\psi)-18)}{108\psi}.\end{aligned}$$

It is easy to show $2\sqrt{3(r\phi(-2c\psi-r\phi+9))}-(9-2c\psi)-18 < 0$. Define

$$\hat{g}(r) = 2\sqrt{3(r\phi(-2c\psi-r\phi+9))}+(9-2c\psi)-18.$$

$\hat{g}(r)$ is concave in r since $\frac{\mathrm{d}^2\hat{g}(r)}{\mathrm{d}r^2} = -\frac{\sqrt{3}\phi^2(9-2c\psi)^2}{2(-r\phi(2c\psi+r\phi-9))^{3/2}} < 0$. Solving for $\hat{g}(r) = 0$, we can get $\hat{g}(r) = 0$ at $r = r^A_{F_t}$ and $r = \hat{r}^A_{F_t}$. Next, we prove Lemma C1 in the following two ranges of c.

(i) When $\frac{9(2-\sqrt{3})}{2\psi} < c \le \frac{9}{2\psi}$, $g_m < 0$, so $S^{A1}_{F_t} < \tilde{\pi}^{AA}_{V_n}$, $S^{A2}_{F_t} < \tilde{\pi}^{AA}_{V_n}$. Additionally, given $\frac{9(2-\sqrt{3})}{2\psi} < c \le \frac{9}{2\psi}$, $r^A_{F_t}$ and $\hat{r}^A_{F_t}$ does not exist, so $\Lambda^{A2}_{F_t}(S^{A1}_{F_t}) > 0$, and $S^{A2}_{F_t} > S^{A1}_{F_t}$.

(ii) When $0 < c \le \frac{9(2-\sqrt{3})}{2\psi}$, $g_m \ge 0$. Next, we consider the following two ranges of r.

(ii-a) When $0 < r \le r^A_{F_t}$, $g(r) \le 0$, so $S^{A1}_{F_t} \le \tilde{\pi}^{AA}_{V_n}$ and $S^{A2}_{F_t} \le \tilde{\pi}^{AA}_{V_n}$. Additionally, $\Lambda^{A2}_{F_t}(S^{A1}_{F_t}) \ge 0$ when $0 < r \le r^A_{F_t}$. Hence, $S^{A1}_{F_t} \le S^{A2}_{F_t}$.

(ii-b) When $r^A_{F_t} < r \le r_m$, $g(r) > 0$, so $S^{A1}_{F_t} > \tilde{\pi}^{AA}_{V_n}$ and $S^{A2}_{F_t} > \tilde{\pi}^{AA}_{V_n}$. Additionally, $\Lambda^{A2}_{F_t}(S^{A1}_{F_t}) < 0$ when $r^A_{F_t} < r \le r_m$. Hence, $S^{A1}_{F_t} > S^{A2}_{F_t}$.

LEMMA C2. *(i)* $S^A_{F_n} > \frac{3(2-\sqrt{3})}{2\psi}$ *if* $S^U_{F_t} > \frac{3(2-\sqrt{3})}{2\psi}$. *(ii)* $S^A_{F_n} > S^U_{F_t}$ *if* $S^U_{F_t} < \frac{3(2-\sqrt{3})}{2\psi}$.

C.2. Proof of Lemma C2.

Define

$$G(r)=S_{F_t}^U-\frac{3(2-\sqrt{3})}{2\psi}=\frac{2\phi r(9+2\phi\Delta q+\phi r-2\psi c)-81(2-\sqrt{3})}{54\psi}.$$

$G(r)$ is convex in r and $G(r)$ gets the minimum value when $r=\frac{-(9+2\phi\Delta q-2\psi c)}{2\phi}<0$. Additionally, When $r=0$,

$$G(0)=\frac{3(-2+\sqrt{3})}{2\psi}<0.$$

Therefore, there must exist a unique

$$r_{ST}=\frac{\sqrt{(4B\phi^2-4c\psi\phi+18\phi)^2-8\left(81\sqrt{3}-162\right)\phi^2}-4B\phi^2+4c\psi\phi-18\phi}{4\phi^2}>0$$

such that $G(r_{ST})=0$. Hence, when $0<r\leq min(r_{ST},r_m)$ (defined in Equation (B.10)), $S_{F_t}^U\leq\frac{3(2-\sqrt{3})}{2\psi}$; otherwise, $S_{F_t}^U>\frac{3(2-\sqrt{3})}{2\psi}$ when $min(r_{ST},r_m)<r\leq r_m$. We then investigate Lemma C2 (i) and (ii) in the following two cases.

(i) When $min(r_{ST},r_m)<r\leq r_m$, $S_{F_t}^U>\frac{3(2-\sqrt{3})}{2\psi}$, so we need to show $S_{F_n}^A>\frac{3(2-\sqrt{3})}{2\psi}$. We can show $S_{F_n}^A$ increases with r since $\frac{\mathrm{d}S_{F_n}^A}{\mathrm{d}r}=-\frac{\partial\Lambda_{F_n}^A/\partial r}{\partial\Lambda_{F_n}^A/\partial S_{F_n}^A}=-\frac{\phi(9-2\phi(\Delta q+r)+2\psi c)}{27\phi(\partial\Lambda_{F_n}^A/\partial S_{F_n}^A)}>0$. When $r=r_m$ and $S=\frac{3(2-\sqrt{3})}{2\psi}$, we have

$$\Lambda_{F_n}^A=\begin{cases}\frac{9\left(3\sqrt{3}-5\right)}{4\psi}-\frac{(\sqrt{3}-3)c}{2}>0 & \text{if } r_m=\frac{9+2\psi c-2\phi\Delta q}{2\phi},\\ \frac{-4(B\phi-2c\psi)^2-54\left(\sqrt{3}-3\right)c\psi+243\left(3\sqrt{3}-5\right)}{108\psi}>0 & \text{if } r_m=\frac{9-2\psi c}{2\phi}.\end{cases}$$

Therefore, $S_{F_n}^A>\frac{3(2-\sqrt{3})}{2\psi}$ when $r=r_m$, and there must exist a

$$r_{st}=\frac{9-2B\phi+2c\psi-3\sqrt{3}\sqrt{9\left(3\sqrt{3}-5\right)-2\left(\sqrt{3}-3\right)c\psi}}{2\phi}$$

such that $S_{F_n}^A=\frac{3(2-\sqrt{3})}{2\psi}$. When $0<r\leq max(0,r_{st})$, $S_{F_n}^A\leq\frac{3(2-\sqrt{3})}{2\psi}$; otherwise, $S_{F_n}^A>\frac{3(2-\sqrt{3})}{2\psi}$ when $max(0,r_{st})<r\leq r_m$. Next, we compare r_{ST} and r_{st}, we can show $r_{ST}>r_{st}$. Hence, if $S_{F_t}^U>\frac{3(2-\sqrt{3})}{2\psi}$, we have $S_{F_n}^A>\frac{3(2-\sqrt{3})}{2\psi}$.

(ii) When $0<r\leq min(r_{ST},r_m)$, $S_{F_t}^U\leq\frac{3(2-\sqrt{3})}{2\psi}$, so we need to show $S_{F_n}^A>S_{F_t}^U$. In what follows, we first show $S_{F_n}^A$ and $S_{F_t}^U$ have exactly one intersection when $r>0$. When $r=0$, we have

$$\Lambda_{F_n}^A(S_{F_t}^U)=-\frac{(9-2\phi\Delta q+2\psi c)^2}{108\psi}<0.$$

Thus, $S_{F_t}^U<S_{F_n}^A$ when $r=0$. Next, $S_{F_t}^U$ is convex increasing in r since $\frac{\mathrm{d}^2S_{F_t}^U}{\mathrm{d}r^2}=\frac{2\phi^2}{27\psi}>0$ and $\frac{\mathrm{d}S_{F_t}^U}{\mathrm{d}r}=\frac{\phi(9+2\phi(\Delta q+r)-2\psi c)}{27\psi}>0$, while $S_{F_n}^A$ is concave increasing in r since $\frac{\mathrm{d}^2S_{F_n}^A}{\mathrm{d}r^2}<0$ and $\frac{\mathrm{d}S_{F_n}^A}{\mathrm{d}r}>0$. Hence, $S_{F_n}^A$ and $S_{F_t}^U$ have exactly one intersection when $r>0$. Moreover, when $r=min(r_{ST},r_m)$, we can show $S_{F_n}^A>\frac{3(2-\sqrt{3})}{2\psi}\geq S_{F_t}^U$. Hence, when $0<r\leq min(r_{ST},r_m)$, $S_{F_n}^A>S_{F_t}^U$. □

Lemma C3. *(i)* $S_{F_t}^U<S_{F_n}^U$ *if* $S_{F_n}^U<\frac{1}{\psi}$; *(ii)* $S_{F_n}^U>\frac{1}{\psi}$ *if* $S_{F_t}^U>\frac{1}{\psi}$.

C.3. Proof of Lemma C3.

First, we find conditions under which $S^U_{F_n} < \frac{1}{\psi}$ and $S^U_{F_n} > \frac{1}{\psi}$, respectively. Then we compare $S^U_{F_t}$ with $min(S^U_{F_n}, \frac{1}{\psi})$. Plug $S = \frac{1}{\psi}$ into $\Lambda^U_{F_n}$, we can get

$$\Lambda^U_{F_n}(\tfrac{1}{\psi}, r) = \frac{-(9-2\phi\Delta q+2\psi\Delta q)^2+54(-3+2\phi r+2\psi c)}{108\psi}.$$

$\Lambda^U_{F_n}(\frac{1}{\psi}, r)$ increases with r. Define

$$r_{AU} = \frac{243-36\phi\Delta q+4\phi^2\Delta q^2-72\psi c-8\phi\psi\Delta qc+4\psi^2c^2}{108\phi}$$

as the solution to $\Lambda^U_{F_n}(\frac{1}{\psi}, r_{AU}) = 0$. Recall from the proof of Lemma 1, when $r \le r_{AU}$, $\frac{1}{\psi} \ge S^U_{F_n}$, and when $r > r_{AU}$, $\frac{1}{\psi} < S^U_{F_n}$. Next, compare $S^U_{F_t}$ with $min(S^U_{F_n}, \frac{1}{\psi})$ in the following three cases.

(i) If $r_{AU} < 0$, $\frac{1}{\psi} < S^U_{F_n}$ when $0 < r \le r_m$ (see Equation (B.10)). Then we compare the value of $\frac{1}{\psi}$ and $S^U_{F_t}$. From the proof of Proposition 1, we have $S^U_{F_t} > \frac{1}{\psi}$ when $r > r_{au} > 0 > r_{AU}$. Hence, if $S^U_{F_t} > \frac{1}{\psi}$, $S^U_{F_n} > \frac{1}{\psi}$.

(ii) If $r_{AU} > r_m$, $\frac{1}{\psi} > S^U_{F_n}$ when $0 < r \le r_m$. So compare the value of $S^U_{F_n}$ and $S^U_{F_t}$. First, we show that there is at most one intersection between $S^U_{F_n}$ and $S^U_{F_t}$ when $r > 0$. When $r = 0$, $S^U_{F_t} = 0$, and

$$S^U_{F_n} = \frac{-16\Delta q^2\phi^2+16\Delta q\phi(9+2c\psi)+5(9+2c\psi)^2+3\sqrt{(9+2c\psi)^2(-32\Delta q^2\phi^2+32\Delta q\phi(9+2c\psi)+(9+2c\psi)^2)}}{2592\psi} > 0.$$

Recall that $S^U_{F_t}$ is convex increasing r when $r > 0$ since $\frac{\mathrm{d}S^U_{F_t}}{\mathrm{d}r} = \frac{\phi(9+2\phi(\Delta q+r)-2\psi c)}{27\psi} > 0$, and $\frac{\mathrm{d}S^U_{F_n}}{\mathrm{d}r} = -\frac{\partial\Lambda^U_{F_n}/\partial r}{\partial\Lambda^U_{F_n}/\partial S^U_{F_n}} = -\frac{\sqrt{S}r/\sqrt{\psi}}{\partial\Lambda^U_{F_n}/\partial S^U_{F_n}} > 0$. Define

$$\tau = \frac{\mathrm{d}S^U_{F_t}}{\mathrm{d}r} - \frac{\mathrm{d}S^U_{F_n}}{\mathrm{d}r} = \frac{\phi(2\phi(\Delta q+r)-2c\psi+9)}{27\psi} + \frac{4S^U_{F_n}\phi}{2c\psi+2r\phi-24\sqrt{S^U_{F_n}\psi}+9}.$$

We can show that $\tau > 0$ given $S^U_{F_n}$ satisfying $0 < S^U_{F_n} < \frac{(9+2\phi r+2\psi c)^2}{144\psi}$ (See the proof of Lemma 3). Therefore, there is at most one intersection between $S^U_{F_n}$ and $S^U_{F_t}$ when $r > 0$. Next, Let $r = r_m$ (see Equation (B.10)) and plug $S^U_{F_t}$ into $\Lambda^U_{F_n}$, we can show that $\Lambda^U_{F_n}(S^U_{F_t}) > 0$. Hence, there is no intersection between $S^U_{F_n}$ and $S^U_{F_t}$ when $0 < r \le r_m$, and we have $S^U_{F_n} > S^U_{F_t}$ if $0 < r \le r_m$.

(iii) If $0 < r_{AU} \le r_m$, we consider the following two subcases.

(iii-a) $\frac{1}{\psi} < S^U_{F_n}$ when $r_{AU} < r \le r_m$. Similarly to Case (i), $\frac{1}{\psi} < S^U_{F_t}$ when $r > r_{au}$. Comparing r_{au} and r_{AU}, we can show that $r_{au} > r_{AU}$, which indicates that $S^U_{F_n} > \frac{1}{\psi}$ if $S^U_{F_t} > \frac{1}{\psi}$.

(iii-b) $\frac{1}{\psi} > S^U_{F_n}$ when $0 < r \le r_{AU}$. Similarly to case (ii), Let $r = r_{AU}$ and plug $S^U_{F_t}$ into $\Lambda^U_{F_n}$, we can show that $\Lambda^U_{F_n}(S^U_{F_t}) > 0$. Hence, there is no intersection between $S^U_{F_n}$ and $S^U_{F_t}$ when $0 < r \le r_{AU}$, and we have $S^U_{F_n} > S^U_{F_t}$ if $0 < r \le r_{AU}$. □

Lemma C4. *$S^U_{F_t} \ge \tilde{\pi}^{UA}_{V_n}$ when*

$$0 < c \le \frac{6\sqrt{6}-9}{2\psi}, \max\left(\frac{\psi c}{\phi}, \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}\right) < \Delta q \le \frac{10c\psi+\sqrt{3}\sqrt{(9-2c\psi)(2c\psi+63)}-9}{8\phi} \text{ and } r_d \ge r \le r_m,$$

where r_d is defined in Equation (C.1).

C.4. Proof of Lemma C4.

Define $\tilde{O} = S_{F_t}^U - \tilde{\pi}_{V_n}^{UA}$. $\tilde{O}$ is convex in r since $\frac{\mathrm{d}^2\tilde{O}}{\mathrm{d}r^2} = \frac{4\phi^2}{81\psi} > 0$. Let $r=0$, and we have $\tilde{O} = -\frac{(2c\psi+9)^2}{324\psi} < 0$. Hence, when $r>0$, $\tilde{O}(r)=0$ has at most one solution. Let $r = r_m$, and if $\tilde{O}(r_m) < 0$, $S_{F_t}^U < \tilde{\pi}_{V_n}^{UA}$; otherwise, if $\tilde{O}(r_m) > 0$, there must exist a

$$r_d = \frac{-18+6\phi\Delta q+8\psi c+\sqrt{6(81+6\phi^2\Delta q^2+4\phi\Delta q(9-4\psi c)+12\psi c(-3+\psi c))}}{4\phi} \tag{C.1}$$

such that $S_{F_t}^U(r_d) = \tilde{\pi}_{V_n}^{UA}(r_d)$, and $r_d < r_m$. We consider the following two cases.

(i) When $\frac{\psi c}{\phi} < \Delta q \le \frac{2\psi c}{\phi}$, $r_m = \frac{9-2\psi c}{2\phi}$. Plug r_m into $\tilde{O}$, we have

$$\tilde{O}(r_m, \Delta q) = \frac{4\Delta q\phi(9-2c\psi)+3(2c\psi-15)(2c\psi-3)}{108\psi}.$$

$\tilde{O}(r_m, \Delta q)$ increases with Δq since $\frac{\partial \tilde{O}(r_m,\Delta q)}{\partial \Delta q} = \frac{\phi(9-2c\psi)}{27\psi} > 0$. Therefore, there must exist a

$$\Delta\hat{q} = \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}$$

such that $\tilde{O}(r_m, \Delta\hat{q}) = 0$. Next, we can get $\Delta\hat{q} \le \frac{2\psi c}{\phi}$ when $0 < c \le \frac{6\sqrt{6}-9}{2\psi}$. Therefore, when $0 < c \le \frac{6\sqrt{6}-9}{2\psi}$ and $\max\left(\frac{\psi c}{\phi}, \frac{135-108c\psi+12c^2\psi^2}{-36\phi+8c\phi\psi}\right) < \Delta q \le \frac{2\psi c}{\phi}$, $\tilde{O}(r_m, \Delta q) \ge 0$.

(ii) If $\frac{2\psi c}{\phi} < \Delta q \le \frac{9+2\psi c}{2\phi}$, $r_m = \frac{9+2c\psi-2\phi\Delta q}{2\phi}$. Plug r_m into $\tilde{O}$, we have

$$\tilde{O}(r_m, \Delta q) = \frac{40\Delta qc\psi\phi-4\Delta q\phi(4\Delta q\phi+9)-28c^2\psi^2-36c\psi+405}{324\psi}.$$

$\tilde{O}$ is concave in Δq since $\frac{\partial^2\tilde{O}}{\partial\Delta q^2} = -\frac{8\phi^2}{81\psi} < 0$. Therefore, there must exist a

$$\Delta\underline{q} = \frac{10c\psi+\sqrt{3}\sqrt{(9-2c\psi)(2c\psi+63)}-9}{8\phi}$$

such that $\tilde{O}(r_m, \Delta\underline{q}) = 0$. The other solution to $\tilde{O}(r_m, \Delta q) = 0$ is $\Delta q = \frac{10c\psi-\sqrt{3}\sqrt{(9-2c\psi)(2c\psi+63)}-9}{8\phi} < \frac{2\psi c}{\phi}$. Similarly as Lemma B1, we can show that $\frac{2c\psi}{\phi} \le \Delta\underline{q} < \frac{9+2\psi c}{2\phi}$ when $0 < c \le \frac{6\sqrt{6}-9}{2\psi}$. In conclusion, when $0 < c \le \frac{6\sqrt{6}-9}{2\psi}$ and $\frac{2\psi c}{\phi} \le \Delta q \le \frac{10c\psi+\sqrt{3}\sqrt{(9-2c\psi)(2c\psi+63)}-9}{8\phi}$, $\tilde{O}(r_m, \Delta q) \ge 0$. □

Appendix D: Sales-Relevant Government Penalty

In this section, we consider the sales-relevant penalty, where the amount of government penalty is proportional to the sales. Specifically, we assume that the expected government penalty in Equation (**??**) is the penalty per unit product sold by the farm/vendor, and the government penalty punished on the farmer/vendor is

$$\bar{S} = S \cdot D_i(x_t, x_n), \tag{D.1}$$

where $S = \gamma * s$ and $D_i(x_t, x_n)$ is the corresponding demand sold by the farm/vendor. In this case, we simplify our model and assume that the traceable supply chain and non-traceable supply chain are homogeneous, that is, we don't consider the extra production cost of the traceable supply chain ($c = 0$), and additionally, the probability of producing high-quality products (θ_0^H) is also the same for both supply chains ($\theta_t^H = \theta_n^H = \theta_0^H$). Consequently, if the farm does not adulterate, the expected quality of the output will be $Q_0 = q_H\theta_0^H + q_L(1-\theta_0^H)$; if the farm adulterates, the expected quality will be $Q_A = q_H\theta^{max} + q_L(1-\theta^{max})$, and $Q_A > Q_0$. Therefore, given the adulteration strategy of farms, the only difference between the traceable supply chain and the non-traceable supply chain is that the government penalty is allocated differently in the two supply chains: in the traceable supply chain, the farm is punished by the government if the product is adulterated. In contrast, in the non-traceable supply chain, the vendor is punished by the government. This simplified model captures the effect of critical factors: traceability and the government penalty on deterring adulteration. Additionally, in Section 3, we discussed the situation where farms are large-scale farms and have the power to decide the wholesale price. Nevertheless, farms with small-scale are price takers and sell their products to the corresponding retailer with open market prices. Hence, we assume that the wholesale price between the farm and the vendor is as follows:

$$w_i = w_0 Q_i(x_i), \quad i \in \{t, n\},$$

where w_0 is the marginal market price, $Q_i(U) = Q_0$ if the farm did not adulterate, and $Q_i(A) = Q_A$ if the farm chose to adulterate. This form of the wholesale price is commonly used in the existing literature (Mu et al. 2016, Ayvaz-Çavdaroğlu et al. 2021). In summary, the sequence of events is as follows.

Stage I: Each farm simultaneously and individually decides whether to adulterate or not. The uncertain quality of each farm's output is realized according to the farms' adulteration decision.

Stage II: Each vendor inspects the corresponding farm's adulteration behavior, and then decides whether to procure the agricultural products from the upstream farm. If vendors decide to procure the agricultural products from the corresponding farm, they pay the wholesale price contingent on the quality. They then set the retail prices and sell products to consumers, respectively.

Stage III: Consumers buy products according to the average qualities and retail prices of outputs in each supply chain.

Stage IV: The government conducts sampling tests on the products sold in the market. The sales-relevant penalty will be imposed on the traceable farm or the non-traceable vendor if adulteration is caught by the government.

To begin with the analysis, we first characterize the expected profit functions of vendors and farms.

$$\Pi_{V_t}^{x_t x_n} = \pi_{V_t}^{x_t x_n}, \tag{D.2}$$
$$\Pi_{V_n}^{x_t x_n} = max(\pi_{V_n}^{x_t x_n} - D_n(x_t, x_n) S \cdot \mathbb{1}_{x_n = A}, 0), \tag{D.3}$$
$$\Pi_{F_t}^{x_t x_n} = \pi_{F_t}^{x_t x_n} - D_t(x_t, x_n) S \cdot \mathbb{1}_{x_t = A}, \tag{D.4}$$
$$\Pi_{F_n}^{x_t x_n} = \begin{cases} \pi_{F_n}^{x_t x_n} & \text{if } V_n \text{ procures from } F_n, \\ 0 & \text{otherwise,} \end{cases} \tag{D.5}$$

where $\pi_{V_i}^{x_t x_n} = (p_i - w_i) D_i(x_t, x_n)$ and $\pi_{F_i}^{x_t x_n} = w_i D_i(x_t, x_n)$ are the revenue of vendors and farms from selling their corresponding products, respectively. Similar to the base model, we first solve for the subgame equilibrium given farms' adulteration decisions in Stage I. Here, we present the sub-equilibrium results in Scenarios (U,A) and (A,U) in the following Lemma, and results for Scenarios (U,U) and (A,A) are listed in the proof of this Lemma[13].

Lemma D1. *(i) In scenario (U,A), the sub-equilibrium retail prices and profits of players in each supply chain are:*

$$p_n^* = \tfrac{3 + 2(Q_A - Q_0)\phi + (4w_0 Q_A + 2w_0 Q_0 + 4S)\psi}{6\psi}, \qquad p_t^* = \tfrac{3 - 2(Q_A - Q_0)\phi + (2w_0 Q_A + 4w_0 Q_0 + 2S)\psi}{6\psi};$$

[13] The proof of this Lemma and proofs of subsequent propositions are provided in the Supplement E.

$$\Pi_{V_n}^{UA*}=\frac{(3+2(Q_A-Q_0)\phi-2(Q_A-Q_0)w_0\psi-2S\psi)^2}{36\psi},\qquad \Pi_{V_t}^{UA*}=\frac{(3-2(Q_A-Q_0)\phi+2(Q_A-Q_0)w_0\psi+2S\psi)^2}{36\psi};$$

$$\Pi_{F_n}^{UA*}=\frac{w_0Q_A(3+2(Q_A-Q_0)\phi-2(Q_A-Q_0)w_0\psi-2S\psi)}{6},\qquad \Pi_{F_t}^{UA*}=\frac{w_0Q_0(3-2(Q_A-Q_0)\phi+2(Q_A-Q_0)w_0\psi+2S\psi)}{6}.$$

(ii) In scenario (A,U), the sub-equilibrium retail prices and profits of players in each supply chain are:

$$p_n^*=\frac{3-2(Q_A-Q_0)\phi+2w_0(Q_A+2Q_0)\psi}{6\psi},\qquad p_t^*=\frac{3+2(Q_A-Q_0)\phi+2w_0(2Q_A+Q_0)\psi}{6\psi};$$

$$\Pi_{V_n}^{AU*}=\frac{(3-2(Q_A-Q_0)\phi+2(Q_A-Q_0)w_0\psi)^2}{36\psi},\qquad \Pi_{V_t}^{AU*}=\frac{(3+2(Q_A-Q_0)\phi-2(Q_A-Q_0)w_0\psi)^2}{36\psi};$$

$$\Pi_{F_n}^{AU*}=\frac{w_0Q_0(3-2(Q_A-Q_0)\phi+2(Q_A-Q_0)w_0\psi)}{6},\qquad \Pi_{F_t}^{AU*}=\frac{(w_0Q_A-S)(3+2(Q_A-Q_0)\phi-2(Q_A-Q_0)w_0\psi)}{6}.$$

Lemma D1 (i) shows that if the non-traceable farm chooses to adulterate, the vendor, who gets punishment from the government, will set a retail price that increases with the unit penalty S to compensate for the penalty loss. As a result, the demand for the non-traceable products decreases with the unit penalty ($D_n(U,A)=\frac{3+2(Q_A-Q_0)\phi-2(Q_A-Q_0)w_0\psi-2S\psi}{6}$), and the non-traceable farm's profit, in turn, decreases with the government penalty. Recall from the base model, the indirect penalty mechanism takes into effect when the non-traceable farm decreases the wholesale price to ensure the downstream vendor sourcing from him. This extension illustrates a different indirect penalty mechanism, that is, the demand for adulterated non-traceable products decreases with the unit government penalty. As the unit government penalty increases, the profit premium for adulterated products would be smaller than the government penalty, leading to the unadulteration of the non-traceable farm. On the other hand, Lemma D1 (ii) shows the traceable farm is still punished by the government directly, which deters the traceable farm from adulterating when the government penalty surpasses the revenue premium. Lemma E1 characterizes the condition under which the direct/indirect penalty mechanism comes into effect by comparing the sub-equilibrium profit of farms before and after adulteration. With subgame equilibria in stages 2-4, one can readily derive the equilibrium adulteration strategy in Stage 1.

Proposition D1. *There exist thresholds of S such that*

$$(x_t^*,x_n^*)=\begin{cases}(U,U)\ if\ S>max(S_{F_n}^U,S_{F_t}^U),\\(A,U)\ if\ S_{F_n}^A<S\le S_{F_t}^U,\\(U,A)\ if\ S_{F_t}^A<S\le S_{F_n}^U,\\(A,A)\ if\ 0<S\le min(S_{F_n}^A,S_{F_t}^A).\end{cases}$$

Similar to the Theorem 1, both farms adulterate in equilibrium when the government penalty is relatively small; otherwise, neither farm adulterates when the government penalty is relatively large. When the government penalty is moderate, the non-traceable farm adulterates (U,A) when the marginal wholesale price is relatively small, that is, $0<w_0<w_{ua}$, where w_{au} is defined in Equation (E.7). In the non-traceable supply chain, when w_0 is relatively small, the unit cost for the non-traceable vendor after considering the government penalty is low, so the non-traceable vendor earns a certain profit for selling the adulterated products and the indirect penalty does not work for the non-traceable farm. On the other hand, for the traceable farm, a small marginal wholesale price represents unit revenue from selling adulterated products is small, and because of the direct penalty, he would not adulterate unless the government penalty is very small. Additionally, the traceable farm adulterates (A,U) (if exists) when the marginal wholesale price is relatively large, that is $w_{au}<w_0<\hat{w}_{au}$, where w_{au} and $\hat{w}_{au}$ are defined in Equation (E.5) and Equation (E.6) respectively. With a high w_0, the traceable farm's unit revenue after considering the government penalty is high, so the farm adulterates. For the non-traceable supply chain, the non-traceable vendor will set a high retail price to offset the high cost, leading to a low demand for adulterated products. Thus, the non-traceable farm will not adulterate.

Moreover, both (A,U) and (U,A) exist when $max(S^A_{F_n}, S^A_{F_t}) < S < min(S^U_{F_n}, S^U_{F_t})$ (if exists). Different from Section 4, we adopt the Pareto dominance to refine the multiple equilibria as we consider the same production cost and initial quality between two farms, in which the degree of asymmetry is low. A Nash equilibrium is considered payoff dominant if it is Pareto superior to all other Nash equilibria in the game. In our model, a unique equilibrium survives the refinement (See the proof of Lemma E2 for detailed information). In the following part, we will demonstrate how different equilibrium regions are positioned concerning the penalty thresholds and focus on the emergence of the equilibrium (A,U) and (U,A). Figure D.1 shows one possible region of equilibrium (A,U) as Region $\mathbb{P}$ (defined in Equation (E.8)) and one possible region of refined equilibrium (U,A) as Region $\mathbb{R}$ (defined in Equation (E.9)). Then, we will discuss how they can be affected with respect to customers' sensitivity to quality (ϕ). In our model, customers' sensitivity to quality (ϕ) reflects the competitive advantage of the adulterated products.

Proposition D2. *The conditions for the emergence of Region $\mathbb{P}$ and Region $\mathbb{R}$ with respect to ϕ are as follows.*

(i) When $\phi > \frac{3}{2Q_A}$, equilibrium (A,U) in Region $\mathbb{P}$ and equilibrium (U,A) in Region $\mathbb{R}$ emerge, as shown in Figure D.1(a).

(ii) When $\frac{3\left(\sqrt{Q_0\left(Q_A^3-2Q_AQ_0^2+Q_0^3\right)}-Q_AQ_0+Q_0^2\right)}{Q_A^3} < \phi \le \frac{3}{2Q_A}$, equilibrium (A,U) in Region $\mathbb{P}$ emerges, as shown in Figure D.1(b).

(iii) When $0 < \phi \le \frac{3\left(\sqrt{Q_0\left(Q_A^3-2Q_AQ_0^2+Q_0^3\right)}-Q_AQ_0+Q_0^2\right)}{Q_A^3}$, neither Region $\mathbb{P}$ and Region $\mathbb{R}$ emerges, as shown in Figure D.1(c).

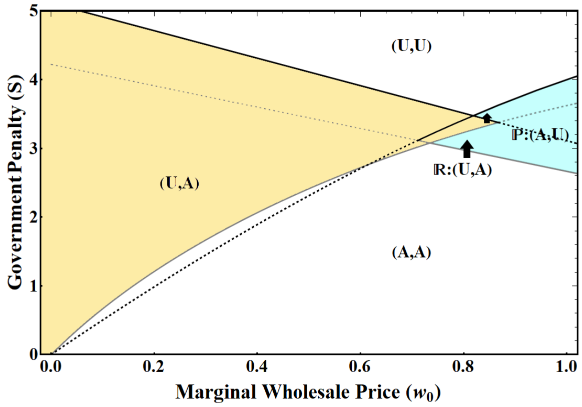
Proposition D2 shows that when customers' sensitivity to quality (ϕ) is large, both Region $\mathbb{P}$ and Region $\mathbb{R}$ emerge. As shown in Figure D.1 (a), the traceable farm adulterates when the government penalty is relatively high (a switch from (U,A) to (A,U)). In addition to the large unit revenue brought by the large marginal wholesale price, a high ϕ provides a large demand for adulterated high-quality products. Hence, the traceable farm's incentive to adulterate given the high government penalty is large. For the non-traceable farm, a large marginal wholesale price leads to a price disadvantage for the non-traceable products, so the non-traceable farm does not adulterate. As ϕ decreases, the demand brought by the adulterated products decreases, so the traceable farm chooses to adulterate when the marginal wholesale price is very large to as the unit revenue at this time is large. Moreover, when ϕ is very small, the traceable farm will stop adulterating when the government penalty surpasses a small threshold, i.e. (A,U) does not exit.

Appendix E: Proof of Section D

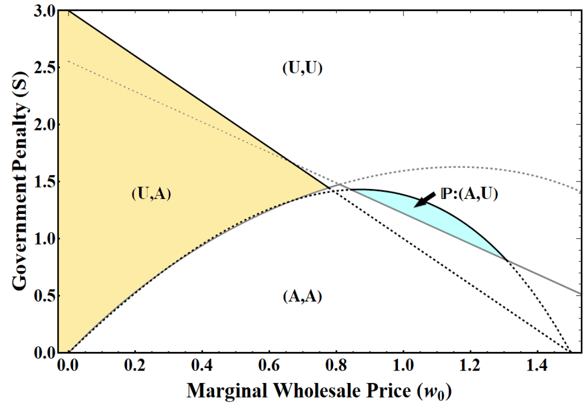
E.1. Proof of Lemma D1.

Given the adulteration decisions of the farms, it is straightforward to show the concavity of each vendor's profit $\Pi^{x_t x_n}_{V_i}$ on p_i, $i \in \{t, n\}$, and the optimal retail prices of scenarios (A,A) and (U,A) are listed in Lemma

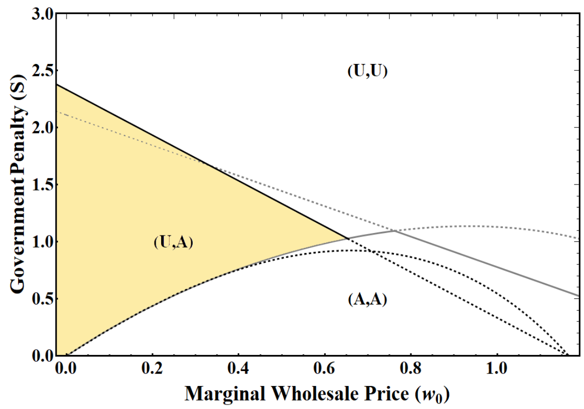
Figure D.1: How the Government Penalty and Quality Enhancement Level Impact the Equilibrium Position



(a) $\phi = 0.6$



(b) $\phi = 0.2$



(c) $\phi = 0.1$

Notes. The parameters are $Q_A = 6$, $Q_0 = 4$, $\psi = 0.3$.

D1. Plug optimal prices into vendors' and farms' profit functions, we can get profits in Lemma D1. Similarly, we solve for the optimal retail prices and profits in scenarios (U,U) and (A,U) and show them in the following:
(i) In scenario (U,U), the sub-equilibrium retail prices and profits of players in each supply chain are:

$$p_n^* = p_t^* = \frac{1+2\psi w_0 Q_0}{2\psi};$$

$$\Pi_{V_n}^{UU*} = \Pi_{V_t}^{UU*} = \frac{1}{4\psi};$$

$$\Pi_{F_n}^{UU*} = \Pi_{F_t}^{UU*} = \frac{w_0 Q_0}{2}.$$

(ii) In scenario (A,A), the sub-equilibrium retail prices and profits of players in each supply chain are:

$$p_n^* = \frac{3+(6w_0Q_A+4S)\psi}{6\psi}, \qquad p_t^* = \frac{3+(6w_0Q_A+2S)\psi}{6\psi};$$

$$\Pi_{V_n}^{AA*} = \frac{(3-2S\psi)^2}{36\psi}, \qquad \Pi_{V_t}^{AA*} = \frac{(3+2S\psi)^2}{36\psi};$$

$$\Pi_{F_n}^{AA*} = \frac{w_0 Q_A(3-2S\psi)}{6}, \qquad \Pi_{F_t}^{AA*} = \frac{(w_0 Q_A - S)(3+2S\psi)}{6}.$$

□

Lemma E1. *(i) Given the traceable farm unadulterating, the non-traceable farm adulterates when* $S < S^U_{F_n}$*, i.e.,* $\Pi^{UA*}_{F_n} > \Pi^{UU*}_{F_n}$*; otherwise, the non-traceable farm does not adulterate.* $S^U_{F_n}$ *is the government penalty threshold satisfying* $\Pi^{UA*}_{F_n} = \Pi^{UU*}_{F_n}$*.*

(ii) Given the non-traceable farm unadulterating, the traceable farm adulterates when $S < S^U_{F_t}$*, i.e.,* $\Pi^{AU*}_{F_t} > \Pi^{UU*}_{F_t}$*; otherwise, the traceable farm does not adulterate.* $S^U_{F_t}$ *is the government penalty threshold satisfying* $\Pi^{AU*}_{F_t} = \Pi^{UU*}_{F_t}$*.*

(iii) Given the traceable farm adulterating, the non-traceable farm adulterates when $S < S^A_{F_n}$*, i.e.,* $\Pi^{AA*}_{F_n} > \Pi^{AU*}_{F_n}$*; otherwise, the non-traceable farm does not adulterate.* $S^A_{F_n}$ *is the government penalty threshold satisfying* $\Pi^{AA*}_{F_n} = \Pi^{AU*}_{F_n}$*.*

(iv) Given the non-traceable farm adulterating, the traceable farm adulterates when $0 < S < S^A_{F_t}$*, i.e.,* $\Pi^{AA*}_{F_t} > \Pi^{UA*}_{F_t}$*; otherwise, the traceable farm does not adulterate.* $S^A_{F_t}$ *is the government penalty threshold satisfying* $\Pi^{AA*}_{F_t} = \Pi^{UA*}_{F_t}$ *and is given in Equation (E.3).*

E.2. Proof of Lemma E1.

To begin with, we characterize conditions under which the demand for each vendor is positive when the government penalty is 0. Let $S = 0$ and plug optimal prices into the demand, we have $D_i(U,U) = D_i(A,A) = \frac{1}{2} > 0$, and

$$D_t(U,A) = D_n(A,U) = \frac{-2(Q_A - Q_0)(\phi - w_0\psi) + 3}{6},$$

which increases in w_0. Define

$$\underline{w} = \frac{-3 + 2(Q_A - Q_0)\phi}{2(Q_A - Q_0)\psi}$$

and we can show that $D_t(U,A) = D_n(A,U) > 0$ when $w_0 > \underline{w}$. Similarly,

$$D_t(A,U) = D_n(U,A) = \frac{2(Q_A - Q_0)(\phi - w_0\psi) + 3}{6},$$

which decreases with w_0. Define

$$w_m = \frac{3 + 2(Q_A - Q_0)\phi}{2(Q_A - Q_0)\psi} \tag{E.1}$$

and we can show that $D_t(A,U) = D_n(U,A) > 0$ when $w_0 < w_m$. Therefore, we consider $w_0 \in (max(0, \underline{w}), w_m)$ in our analysis. In the following, we consider the situation when $\underline{w} < 0$ as the results of $\underline{w} > 0$ are similar to the case when $\underline{w} < 0$.

(i) Define $\Theta(S) = \Pi^{UA*}_{F_n} - \Pi^{UU*}_{F_n}$. $\Theta(S)$ decreases in S since $\frac{\mathrm{d}\Theta(S)}{\mathrm{d}S} = -\frac{w_0 Q_A \psi}{3} < 0$. Hence, there must exist a

$$S^U_{F_n} = \frac{(Q_A - Q_0)(3 + 2Q_A(\phi - w_0\psi))}{2\psi Q_A}$$

such that $\Theta(S) = 0$. When $S < S^U_{F_n}$, $\Pi^{UA*}_{F_n} > \Pi^{UU*}_{F_n}$. On the other hand, demand for non-traceable products decreases with the government penalty in scenario (U,A), i.e., $D_n(U,A) = \frac{-2S\psi + 2(Q_A - Q_0)(\phi - w_0\psi) + 3}{6}$. Hence, when $S < \frac{2(Q_A - Q_0)(\phi - w_0\psi) + 3}{2\psi}$, $D_n(U,A) > 0$. Therefore, when

$$S < min(S^U_{F_n}, \frac{2(Q_A - Q_0)(\phi - w_0\psi) + 3}{2\psi}),$$

the non-traceable farm chooses to adulterate. Moreover, it is easy to show $S^U_{F_n} < \frac{2(Q_A - Q_0)(\phi - w_0\psi) + 3}{2\psi}$, so the above condition can be refined as $S < S^U_{F_n}$. We should note that $S^U_{F_n}$ decreases with w_0 since $\frac{\mathrm{d}S^U_{F_n}}{\mathrm{d}w_0} = -(Q_A - Q_0) < 0$. So there exists a $\hat{w} = \frac{2Q_A\phi + 3}{2Q_A\psi} < w_m$ such that $S^U_{F_n} = 0$, when $0 < w_0 < \hat{w}$, $S^U_{F_n} > 0$.

(ii) Similar to case (i), $\Pi_{F_t}^{AU*} - \Pi_{F_t}^{UU*}$ decreases in S, and

$$S_{F_t}^{U} = \frac{Q_A w_0(3+2(Q_A-Q_0)(\phi-\psi w_0))-3Q_0 w_0}{3+2(Q_A-Q_0)(\phi-\psi w_0)}$$

satisfies $\Pi_{F_t}^{AU*} - \Pi_{F_t}^{UU*} = 0$. On the other hand, if the traceable farm adulterates, there will be a unit penalty on him. So if $S < w_0 Q_A$, the unit revenue for selling the traceable products is larger than 0. Therefore, when

$$S < min(S_{F_t}^{U}, w_0 Q_A),$$

the traceable farm chooses to adulterate. Moreover, it is easy to show $S_{F_t}^{U} < w_0 Q_A$, so the above condition can be refined as $S < S_{F_t}^{U}$. We should note that $S_{F_t}^{U}$ is concave in w_0 since $\frac{\mathrm{d}^2 S_{F_t}^{U}}{\mathrm{d}w_0^2} = -\frac{12\psi Q_0(Q_A-Q_0)(2Q_A\phi-2Q_0\phi+3)}{(2(Q_A-Q_0)(\phi-w\psi)+3)^3} < 0$ when $w_0 < w_m$. Solving for $S_{F_t}^{U} = 0$, we have $S_{F_t}^{U} > 0$ when $0 < w_0 < \hat{w}$.

(iii) Similar to case (i), $\Pi_{F_n}^{AA*} - \Pi_{F_n}^{AU*}$ decreases in S, and

$$S_{F_n}^{A} = \frac{(Q_A-Q_0)(3+2Q_0(\phi-\psi w_0))}{2\psi Q_A}$$

satisfies $\Pi_{F_n}^{AA*} = \Pi_{F_n}^{AU*}$. On the other hand, demand for non-traceable products decreases with the government penalty in scenario (A,A), i.e., $D_n(A,A) = \frac{3-2S\psi}{6}$. Hence, when $S < \frac{3}{2\psi}$, $D_n(U,A) > 0$. Therefore, when

$$S < min(S_{F_n}^{A}, \tfrac{3}{2\psi}),$$

the non-traceable farm chooses to adulterate. Moreover, it is easy to show $S_{F_n}^{A} < \frac{3}{2\psi}$, so the above condition can be refined as $S < S_{F_n}^{A}$. We should note that $S_{F_n}^{A}$ decreases with w_0 since $\frac{\mathrm{d}S_{F_n}^{A}}{\mathrm{d}w_0} = -\frac{Q_0(Q_A-Q_0)}{Q_A} < 0$. So there exists a $\tilde{w} = \frac{2Q_0\phi+3}{2\psi Q_0} < w_m$ such that $S_{F_n}^{A} = 0$, when $0 < w_0 < \tilde{w}$, $S_{F_n}^{A} > 0$.

(iv) Define $\hat{\Theta}(S) = \Pi_{F_t}^{AA*} - \Pi_{F_t}^{UA*}$. $\hat{\Theta}(S)$ is concave in S since $\frac{\mathrm{d}^2\hat{\Theta}(S)}{\mathrm{d}S^2} = -\frac{2\psi}{3} < 0$. Then we find conditions under which $\hat{\Theta}(S) = 0$ has solutions. To start, define $S_e = \frac{2w_0\psi(Q_A-Q_0)-3}{4\psi}$ satisfying $\frac{\mathrm{d}\hat{\Theta}}{\mathrm{d}S} = 0$. Plug S_e into $\hat{\Theta}$, we have

$$\hat{\Theta}(S_e) = \frac{4w_0\psi(Q_A-Q_0)(w_0\psi(Q_A-5Q_0)+4Q_0\phi+3)+9}{48\psi}.$$

We can show that $\hat{\Theta}(S_e)$ is smaller than 0 when

$$Q_A < 5Q_0 \text{ and } w_e < w_0 < w_m (\text{Condition A}). \tag{E.2}$$

where $w_e = \frac{9}{4\sqrt{\psi^2 Q_0(Q_A-Q_0)(9+2\phi(Q_A-Q_0)(2Q_0\phi+3))}-2\psi(Q_A-Q_0)(4Q_0\phi+3)}$. Therefore, when Condition A is satisfied, $\hat{\Theta}(S) = 0$ does not exist; otherwise, $\hat{\Theta}(S) = 0$ has two solutions. Hence, define two thresholds

$$S_{F_t}^{A} = \frac{-3+2\psi w_0(Q_A-Q_0)+\sqrt{9+4\psi w_0(Q_A-Q_0)(3+4\phi Q_0+\psi w_0 Q_A-5\psi w_0 Q_0)}}{4\psi}, \tag{E.3}$$

$$S_{F_t}^{A2} = \frac{-3+2\psi w_0(Q_A-Q_0)-\sqrt{9+4\psi w_0(Q_A-Q_0)(3+4\phi Q_0+\psi w_0 Q_A-5\psi w_0 Q_0)}}{4\psi} \tag{E.4}$$

as the solution to $\hat{\Theta}(S) = 0$ (if exists), $S_{F_t}^{A2} < S_{F_t}^{A}$. Then we consider the following subcases.

(iv-a) If the solution to $\hat{\Theta}(S) = 0$ exists, we have $\hat{\Theta}(S) > 0$ when $S_{F_t}^{A2} < S < S_{F_t}^{A}$. On the other hand, if the traceable farm adulterates, there will be a unit penalty on him. So if $S < w_0 Q_A$, the unit revenue for selling the traceable products is larger than 0. Therefore, when

$$max(0, S_{F_t}^{A2}) < S < min(S_{F_t}^{A}, w_0 Q_A),$$

Table B5: Payoff Matrix for the Farm's Adulteration Choices

		Non-traceable farm's choice	
	$(\Pi_{F_t}^{x_t x_n *}, \Pi_{F_n}^{x_t x_n *})$	A	U
Traceable farm's choice	A	$\frac{(Q_A w_0 - S)(3+2\psi S)}{6}, \ \frac{Q_A w_0 (3-2\psi S)}{6}$	$\frac{(Q_A w_0 - S)(3+(2\phi-2\psi w_0)(Q_A - Q_0))}{6}, \frac{Q_0 w_0 (3-(2\phi-2\psi w_0)(Q_A - Q_0))}{6}$
	U	$\frac{Q_0 w_0 (3-(2\phi-2\psi w_0)(Q_A-Q_0)+2S\psi)}{6}, \frac{Q_A w_0 (3+(2\phi-2\psi w_0)(Q_A-Q_0)-2S\psi)}{6}$	$\frac{wQ_0}{2}, \ \frac{wQ_0}{2}$

the traceable farm chooses to adulterate. Moreover, it is easy to show $S_{F_t}^A < w_0 Q_A$, so the above condition can be refined as $max(0, S_{F_t}^{A2}) < S < S_{F_t}^A$. Similar to case (i), (ii), and (iii), we can show that given the existence of $S_{F_t}^A$ and $S_{F_t}^{A2}$, $S_{F_t}^{A2} < 0$ and $S_{F_t}^A > 0$ when $0 < w_0 < \tilde{w}$.

(iv-b) If the solution to $\hat{\Theta}(S) = 0$ does not exist, we have $\hat{\Theta}(S) < 0$ when $S > 0$, so the traceable farm does not adulterate, given the non-traceable farm adulterates.

Combining (i), (ii), (iii), and (iv), we further assume $0 < w_0 < \hat{w}$ ($\hat{w} < \tilde{w} < min(w_e, w_m)$)to avoid the trivial cases where the penalty thresholds are smaller than 0. We should note that all four cases exist in this region.

□

E.3. Proof of Proposition D1

Similar to Theorem 1, (x_t^*, x_n^*) is the nash equilibirum if and only if $\Pi_{F_t}^{x_t^* x_n^*} \geq \Pi_{F_t}^{x_t x_n^*}$; and $\Pi_{F_n}^{x_t^* x_n^*} \geq \Pi_{F_n}^{x_t^* x_n}$. Each farm's adulteration strategy and the corresponding payoff are shown in the Table B5, which presents the farms' profits given their adulteration decisions. Then we consider the following four cases.

(i) Scenario (U,U) is the Nash Equilibrium: $\Pi_{F_n}^{UU*} \geq \Pi_{F_n}^{UA*}$ and $\Pi_{F_t}^{UU*} \geq \Pi_{F_t}^{AU*}$.

The condition for $\Pi_{F_n}^{UU*} \geq \Pi_{F_n}^{UA*}$ is $S \geq S_{F_n}^U$, as shown in Lemma E1 (i). The condition for $\Pi_{F_t}^{UU*} \geq \Pi_{F_t}^{AU*}$ is $S \geq S_{F_t}^U$, as shown in Lemma E1 (ii). Hence, the condition for Nash equilibrium (U,U) is $S \geq max(S_{F_n}^U, S_{F_t}^U)$.

(ii) Scenario (A,U) is the Nash equilibrium: $\Pi_{F_n}^{AU*} \geq \Pi_{F_n}^{AA*}$ and $\Pi_{F_t}^{AU*} \geq \Pi_{F_t}^{UU*}$.

The condition for $\Pi_{F_n}^{AU*} \geq \Pi_{F_n}^{AA*}$ is $S \geq S_{F_n}^A$ as shown in Lemma E1 (iii). The condition for $\Pi_{F_t}^{AU*} \geq \Pi_{F_t}^{UU*}$ is $S \leq S_{F_t}^U$, as shown in Lemma E1 (ii). Hence, the condition for Nash equilibrium (A,U) is $S_{F_n}^A \leq S \leq S_{F_t}^U$.

(iii) Scenario (U,A) is the Nash equilibrium: $\Pi_{F_n}^{UA*} \geq \Pi_{F_n}^{UU*}$ and $\Pi_{F_t}^{UA*} \geq \Pi_{F_t}^{AA*}$.

The condition for $\Pi_{F_n}^{UA*} \geq \Pi_{F_n}^{UU*}$ is $S \leq S_{F_n}^U$ as shown in Lemma E1 (i). The condition for $\Pi_{F_t}^{UA*} \geq \Pi_{F_t}^{AA*}$ is $S \geq S_{F_t}^A$ as shown in Lemma E1 (iv). Hence, the condition for Nash equilibrium (U,A) is $S_{F_t}^A \leq S \leq S_{F_n}^U$.

(iv) Scenario (A,A) is the Nash equilibrium: $\Pi_{F_n}^{AA*} \geq \Pi_{F_n}^{AU*}$ and $\Pi_{F_t}^{AA*} \geq \Pi_{F_t}^{UA*}$.

The condition for $\Pi_{F_n}^{AA*} \geq \Pi_{F_n}^{AU*}$ is $S \leq S_{F_n}^A$ as shown in Lemma E1 (iii). The condition for $\Pi_{F_t}^{AA*} \geq \Pi_{F_t}^{UA*}$ is $0 \leq S \leq S_{F_t}^A$ as shown in Lemma E1 (iv). Hence, the condition for Nash equilibrium (A,A) is $0 \leq S \leq min(S_{F_t}^A, S_{F_n}^A)$.

In conclusion, farm's adulteration strategies in equilibrium with respect to the government penalty are listed as follows:

$$(x_t^*, x_n^*) = \begin{cases} (U,U) \ if \ S > max(S_{F_n}^U, S_{F_t}^U), \\ (A,U) \ if \ S_{F_n}^A < S \leq S_{F_t}^U, \\ (U,A) \ if \ S_{F_t}^A < S \leq S_{F_n}^U, \\ (A,A) \ if \ 0 < S \leq min(S_{F_n}^A, S_{F_t}^A). \end{cases}$$

Since $min(S_{F_n}^A, S_{F_t}^A) > 0$ and $max(S_{F_n}^U, S_{F_t}^U) > 0$, equilibria (A,A) and (U,U) always exist. While the existence of equilibria (A,U) and (U,A) is uncertain. In the following, we will find conditions for the existence of equilibria (A,U) and (U,A), respectively.

(i) Define $Q = S_{F_n}^A - S_{F_t}^U$, Q is convex in w since $\frac{\mathrm{d}^2 Q}{\mathrm{d}w^2} = \frac{12\psi Q_0(Q_A-Q_0)(2Q_A\phi-2Q_0\phi+3)}{(2(Q_A-Q_0)(\phi-w\psi)+3)^3} > 0$. Then we solve for $Q=0$, and get the following two subcases.

(i-a) If $\phi > \frac{3\left(\sqrt{Q_0\left(Q_A^3-2Q_AQ_0^2+Q_0^3\right)}-Q_AQ_0+Q_0^2\right)}{Q_A^3}$, $S_{F_n}^A < S_{F_t}^U$ when $w_{au} < w_0 < \hat{w}_{au}$, where

$$w_{au} = \frac{-\sqrt{Q_A^4\phi^2+6Q_AQ_0\phi(Q_A-Q_0)+9Q_0(Q_0-Q_A)}+\phi\left(Q_A^2+2Q_AQ_0-2Q_0^2\right)+3Q_A}{2\psi\left(Q_A^2+Q_AQ_0-Q_0^2\right)}, \tag{E.5}$$

$$\hat{w}_{au} = \frac{\sqrt{Q_A^4\phi^2+6Q_AQ_0\phi(Q_A-Q_0)+9Q_0(Q_0-Q_A)}+\phi\left(Q_A^2+2Q_AQ_0-2Q_0^2\right)+3Q_A}{2\psi\left(Q_A^2+Q_AQ_0-Q_0^2\right)}. \tag{E.6}$$

(i-b) If $\phi < \frac{3\left(\sqrt{Q_0\left(Q_A^3-2Q_AQ_0^2+Q_0^3\right)}-Q_AQ_0+Q_0^2\right)}{Q_A^3}$, $S_{F_n}^A > S_{F_t}^U$.

(ii) Define $\hat{Q} = S_{F_t}^A - S_{F_n}^U$, $\hat{Q}$ is concave in w since $\frac{\mathrm{d}^2\hat{Q}}{\mathrm{d}w^2} = \frac{4\psi Q_0(Q_A-Q_0)(-2\phi(Q_A-Q_0)(2Q_0\phi+3)-9)}{(4w\psi(Q_A-Q_0)(wQ_A\psi-5w\psi Q_0+4Q_0\phi+3)+9)^{3/2}} < 0$. Then we solve for $\hat{Q}=0$, and get $S_{F_t}^A < S_{F_n}^U$ when $w < w_{ua}$ (The other solution to $\hat{Q}=0$ is larger than $\hat{w}$, thus is eliminated.), where

$$w_{ua} = -\frac{\sqrt{4Q_A^4\phi^2+12Q_A\phi(Q_A-Q_0)(3Q_A-2Q_0)+9(Q_A-Q_0)(9Q_A-5Q_0)}+Q_A(-6Q_A\phi+4Q_0\phi-15)+9Q_0}{4Q_A\psi(2Q_A-Q_0)}. \tag{E.7}$$

In summary, equilibrium (A,U) exists when $\phi > \frac{3\left(\sqrt{Q_0\left(Q_A^3-2Q_AQ_0^2+Q_0^3\right)}-Q_AQ_0+Q_0^2\right)}{Q_A^3}$, and $w_{au} < w_0 < \hat{w}_{au}$; equilibrium (U,A) exists when $w < w_{ua}$. Furthermore, we find that given the existence of (A,U) and (U,A), when $\phi > \frac{3}{2Q_A}$ ($\frac{3}{2Q_A} > \frac{3\left(\sqrt{Q_0\left(Q_A^3-2Q_AQ_0^2+Q_0^3\right)}-Q_AQ_0+Q_0^2\right)}{Q_A^3}$), both equilibria (A,U) and (U,A) exist when

$$max(S_{F_n}^A, S_{F_t}^A) < S < min(S_{F_n}^U, S_{F_t}^U).$$

From Lemma E2, we can show equilibrium (U,A) always dominates (A,U). □

LEMMA E2. *(i) Equilibrium (U,A) Pareto dominates (A,U) when* $max(S_{F_n}^A, S_{F_t}^A) < S < min(S_{F_n}^U, S_{F_t}^U)$ *(if exists). (ii) Equilibrium (A,A) Pareto dominates (U,U) when* $max(S_{F_n}^U, S_{F_t}^U) < S < min(S_{F_n}^A, S_{F_t}^A)$.

E.4. Proof of Lemma E2.

Moreover, with the existence of (A,U) and (U,A), we find that both (A,U) and (U,A) exist when

$$max(S_{F_n}^A, S_{F_t}^A) < S < min(S_{F_n}^U, S_{F_t}^U) \text{ (if exists).}$$

Additionally, under some conditions, we have $S_{F_n}^U < S_{F_t}^U < min(S_{F_n}^A, S_{F_t}^A)$. Hence, multiple equilibria (U,U) and (A,A) exist when

$$max(S_{F_n}^U, S_{F_t}^U) < S < min(S_{F_n}^A, S_{F_t}^A).$$

To resolve the issues of multiple equilibria, we adopt the concept of Pareto dominance. A Nash equilibrium is considered payoff dominant if it is Pareto superior to all other Nash equilibria in the game. For (U,A) to Pareto dominate (A,U), mathematically, it is given by

$$\Pi_{F_n}^{UA*} > \Pi_{F_n}^{AU*} \text{ and } \Pi_{F_t}^{UA*} > \Pi_{F_t}^{AU*}.$$

Comparing profits $\Pi_{F_n}^{UA*}$, $\Pi_{F_n}^{AU*}$, $\Pi_{F_t}^{UA*}$ and $\Pi_{F_t}^{AU*}$, we have $\Pi_{F_n}^{UA*} > \Pi_{F_n}^{AU*}$ and $\Pi_{F_t}^{UA*} > \Pi_{F_t}^{AU*}$ when $\hat{S}_l < S < \hat{S}_h$. $\hat{S}_l$ and $\hat{S}_h$ are defined in the following equations.

$$\hat{S}_l = \frac{w_0(Q_A-Q_0)(2(Q_A+Q_0)(\phi-w_0\psi)+3)}{2(6Q_A(\phi-w_0\psi)+7w_0\psi Q_0-6Q_0\phi+9)},$$

$$\hat{S}_h = \frac{(Q_A - Q_0)(2(Q_A+Q_0)(\phi - w_0\psi)+3)}{2Q_A\psi}.$$

Moreover, we can show $\hat{S}_l < max(S^A_{F_n}, S^A_{F_t})$ and $\hat{S}_h > min(S^U_{F_n}, S^U_{F_t})$. Hence, for any $max(S^A_{F_n}, S^A_{F_t}) < S < min(S^U_{F_n}, S^U_{F_t})$, we have equilibrium (U,A) dominates equilibrium (U,A).

Similarly, for (A,A) to Pareto dominate (U,U), the profit for both traceable farm and non-traceable farm in (A,A) should be larger than that in (U,U). Mathematically, it is given by

$$\Pi^{AA*}_{F_n} > \Pi^{UU*}_{F_n} \text{ and } \Pi^{AA*}_{F_t} > \Pi^{UU*}_{F_t}.$$

Comparing profits $\Pi^{AA*}_{F_n}$, $\Pi^{UU*}_{F_n}$, $\Pi^{AA*}_{F_t}$ and $\Pi^{UU*}_{F_t}$, we have $\Pi^{AA*}_{F_n} > \Pi^{UU*}_{F_n}$ and $\Pi^{AA*}_{F_t} > \Pi^{UU*}_{F_t}$ when $S < min(S_l, S_h)$. S_l and S_h are defined in the following equations.

$$S_l = \frac{2w_0Q_A\psi + \sqrt{4w_0\psi(Q_A(w_0Q_A\psi+3)-6Q_0)+9}-3}{4\psi},$$

$$S_h = \frac{3Q_A - 3Q_0}{2Q_A\psi}.$$

Moreover, we can show $min(S_l, S_h) > min(S^A_{F_n}, S^A_{F_t})$. Hence, for any $max(S^U_{F_n}, S^U_{F_t}) < S < min(S^A_{F_n}, S^A_{F_t})$, we have equilibrium (A,A) dominates equilibrium (U,U). □

E.5. Proof of Proposition D2.

Define

$$\mathbb{R} = \{S : max(S^A_{F_n}, S^A_{F_t}) < S < min(S^U_{F_n}, S^U_{F_t})\}, \tag{E.8}$$

$$\mathbb{P} = \mathbb{P}_1 - \mathbb{R}, \tag{E.9}$$

where $\mathbb{P}_1 = \{S : S^A_{F_n} < S < S^U_{F_t}\}$.

From the proof of Proposition D1, Region $\mathbb{R}$ emerges when $\phi > \frac{3}{2Q_A}$, Region $\mathbb{P}$ emerges when $\phi > \frac{3\left(\sqrt{Q_0\left(Q_A^3 - 2Q_AQ_0^2 + Q_0^3\right)} - Q_AQ_0 + Q_0^2\right)}{Q_A^3}$, and $\frac{3}{2Q_A} > \frac{3\left(\sqrt{Q_0\left(Q_A^3 - 2Q_AQ_0^2 + Q_0^3\right)} - Q_AQ_0 + Q_0^2\right)}{Q_A^3}$. Hence, we have our results in Proposition D2. □

References

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