Online Food Delivery Contracting in Three-Sided Markets

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Problem definition: We examine a three-sided food delivery market in which an online food delivery platform should match customers' online orders and self-scheduling delivery drivers. The platform also needs to manage its relationship with a restaurant that provides food through this platform while also offering an alternative dine-in option. Different contracting schemes governing the relationship between the platform and the restaurant affect their profitability. ***Methodology/results:*** We develop a game-theoretic model to investigate the contracting strategies of the platform and the restaurant under three prevalent contracts: dynamic-price/dynamic-wage, fixed-price/dynamic-wage, and dynamic-price/fixed-wage contracts. We show that the price competition between the online and dine-in channels is more fierce in the sharing economy compared to the traditional economy (with fixed labor supply) if and only if the fixed supply is more than that in the sharing economy, regardless of the contracting scheme. Although all contracts lead to the same market outcome in the traditional economy, self-scheduling drivers significantly influence the performance of these contracts in the sharing economy. The dynamic-price/fixed-wage contract induces the most fierce competition in the food market, while the dynamic-price/dynamic-wage contract results in the softest. The platform prefers the fixed-price/dynamic-wage contract, while other parties in the food delivery market usually prefer the dynamic-price/fixed-wage contract. Moreover, we show that the contractual relationship in the food delivery market does not affect the dine-in offline prices, which supports the observation of the restaurant's relatively robust dine-in prices. ***Managerial implications:*** Despite its prevalence, the dynamic-price/dynamic-wage contract typically results in the poorest performance for the platform and moderate performance for the restaurant. Unless the supply is excessively costly, a dynamic-price/fixed-wage contract with well-designed subscription fees can benefit all parties in the food delivery chain, including the drivers and customers. Our findings also offer guidance to policymakers in balancing the interests of gig workers and society. A relatively high minimum wage rate (wage per delivery) can harm society and gig workers, while a relatively high minimum wage (per hour) can benefit both.

Key words: three-sided markets; online food delivery platforms; sharing economy

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1. Introduction

Online food delivery platforms have surged in prominence and growth in recent years, particularly during the COVID-19 pandemic. According to the latest report by IMARC Group (2024), the global online food delivery market reached $134.9 billion in 2023 and is projected to expand at a compound annual growth rate of 9.7% from 2024 to 2032. Platforms such as Uber Eats, Postmates, and Deliveroo in the US, and Meituan and Eleme in China, operate as three-sided platforms connecting customers, delivery drivers, and restaurants. Customers can now order food online without traveling to restaurants, while freelance delivery drivers, compensated by the platforms, handle the delivery. This setup provides restaurants with an alternative online sales channel, allowing them to reach a broader customer base and expand their market presence without incurring significant additional operating costs.

Online food delivery markets are part of the sharing economy, where platforms are compensated only when they successfully match customer demand with service providers. Thus, managing this match is crucial for platforms. Unlike in the traditional economy, where service supply is fixed, service providers in the sharing economy are freelance workers with flexible working options, deciding when to work for a platform based on their compensation. In the online food delivery market, service providers are self-scheduling delivery drivers, and their availability is influenced by the wages set by the platform. Consequently, platforms attempt to control supply through wage adjustments. Simultaneously, customer demand is influenced by the online and offline prices set through interactions between the platform and the restaurant. Therefore, platforms must effectively manage their relationship with restaurants to control online customer demand and align it with the availability of drivers.

This paper examines a three-sided food delivery market operating within the sharing economy. We analyze the competition between an online channel, where a restaurant sells food via an online food delivery platform, and an offline channel, where the restaurant serves dine-in customers directly. The platform sets wages on the supply side for self-scheduling delivery drivers and cooperates with the restaurant to set the online channel prices according to the contracting schemes agreed upon between them. Different platforms employ various contracting schemes with restaurants. For instance, major food delivery platforms like DoorDash and Uber Eats offer marketplace plans featuring dynamic-price contracts. They provide restaurants with basic, plus, and premier service tiers, each associated with progressively higher commission fees. These platforms typically set market prices by adding a delivery fee to the restaurant's price (Christopher 2023). Additionally, DoorDash offers another service called DoorDash Storefront, which does not charge restaurants a commission fee. However, in this case, DoorDash charges customers a higher delivery fee on top of the food price (Doordash 2023b). In contrast, leading Chinese food delivery platforms, Meituan and Eleme, employ fixed-price contracts. In these arrangements, restaurants determine the final online food price, including delivery fees or discounts, while the platforms charge a fixed commission fee (Eleme 2024).

Similarly, platforms have implemented various strategies to manage delivery drivers and control labor supply. Specifically, platforms can adjust drivers' wages in the market, thus flexibly regulating the number of delivery drivers. A representative example is the crowdsourcing model used by Uber Eats, DoorDash, Meituam, and Eleme, where crowdsourced drivers are typically part-time workers and casual labor from the local community. Alternatively, platforms can establish more stable arrangements with drivers by offering fixed wages through fixed-wage contracts. Both Meituan and Eleme provide dedicated delivery drivers with fixed hourly wages, distinguishing them from crowdsourced drivers. Additionally, regulators have begun enforcing minimum wage laws to protect gig workers. For example, the New York State Supreme Court has mandated that DoorDash and Grubhub must pay their delivery drivers at least $17.96 per hour or 50 cents per minute of delivery (PYMNTS 2023, Lindeque 2024). This minimum wage functionally acts as a fixed-wage contract, as platforms are usually unwilling to pay more than these minimum wages. With many countries enacting laws to reclassify gig workers as employees, we anticipate that fixed-wage contracts will become increasingly prevalent (Hu and Liu 2023).

Given the various challenges faced by online food delivery platforms, such as government reclassification of gig workers as employees, the increasing number of gig workers choosing to work across multiple sharing economy platforms (due to the self-scheduling nature of their work), and regulatory efforts to enforce minimum wage standards, platforms must strategically manage their interactions with customers, restaurants, and delivery drivers to achieve success. First, platforms should understand how the self-scheduling nature of supply affects market competition between the online and offline channels. Different contractual relationships in these three-sided markets (i.e., the dynamic/fixed-prices/wages contracts) raise questions about the relative performance of these contracting schemes for players involved in such a market, particularly the restaurant, the platform, and the whole food delivery chain. Moreover, given the recent scrutiny of the government and the regulatory body to protect gig workers, it is interesting to study gig workers' welfare under different contracting schemes and to understand better how the regulatory body should deploy its regulations to protect not only gig workers but also the other players in the market and society as a whole.

To answer these questions, we develop a game-theoretic model in which a restaurant serves customers through two competing channels, the dine-in offline and online through a food delivery platform. Customers can either visit the restaurant in person or place orders on the platform, which matches delivery demands with the labor supply of delivery drivers. We assume the customers are sensitive to the prices of the two channels, and the demand of each channel is linear in the prices. Delivery drivers are self-scheduling, and the wages offered by the platform determine their labor supply. We investigate three common contracting schemes in practice: dynamic-price/dynamic-wage (DD) contract, fixed-price/dynamic-wage (FD) contract, and dynamic-price/fixed-wage (DF) contract. By deriving the equilibrium market outcomes for these contracts, we first investigate how the self-scheduling nature of drivers affects the contractual relationship between the platform and the restaurant, as well as the resulting market outcomes. We then study the platform and the restaurant's preference over these contracting schemes by comparing the market outcomes under each contract. Finally, we assess how minimum wage regulations, whether based on hourly rates or per delivery, impact drivers, society, and the overall food delivery chain.

To examine the impact of self-scheduling drivers in the sharing economy on the food delivery market, we first examine a benchmark case in the traditional economy, where driver supply is exogenous. In the benchmark case, all the contracting schemes result in identical market outcomes; that is, online and offline prices and demands are the same, with only the division of profit differing between firms. In contrast, all these contracting schemes yield quite different market outcomes under the sharing economy. Moreover, we illustrate that the traditional economy demonstrates more intense market competition than the sharing economy if the fixed labor supply of the delivery drivers is larger than the endogenously determined supply of drivers in the sharing economy. In other words, the sharing economy would soften market competition only if the platform has an ample labor supply in the traditional economy.

We also find that the DF contract results in the most fierce competition between the online and offline channels. In particular, under relatively high supply costs, the DF contract results in an oversupply in the online channel compared to a centralized scenario. Under the DF contract, commitment to a sufficient supply aligns both firms' incentives to set lower margins in the online channel, thereby intensifying competition between the online and offline channels. This oversupply is unique to the sharing economy. In a competitive two-sided market, Zhang et al. (2022a) and Hu and Liu (2023) show that wage commitment can intensify market price competition only when competition in the supply market is more intense than in the demand market. Our findings may seem similar, but they are in nature. Additionally, we show that, despite its common use, the DD contract results in the softest competition between online and offline channels, leading to reduced online demand for the platform.

The FD contract outperforms the others for the platform, while the DD contract delivers the worst performance unless the supply cost is excessively high. In the FD contract, the platform delegates the online channel pricing to the restaurant but sets its commission first. Since the only lever the platform can use to match labor supply with the online demand in the market (i.e., the last stage) is the delivery wage offered to the drivers, the restaurant has to reduce its margin to induce larger online orders, incentivizing the platform to raise the drivers' supply through higher wages. In contrast, the DD contract allows the platform to adjust both online prices and delivery wages simultaneously. Such flexibility for the platform enables the restaurant to soften competition by raising its margin, which negatively impacts the platform's profitability. The DF contract performs moderately for the platform. Under the DF contract, the platform finds it optimal to boost the labor supply of drivers by committing to a high wage, which motivates the restaurant to charge lower margins in the online channel, expecting competitive prices from the platform in the online channel. However, the increased wage moderates the effect of higher online orders on the platform's profit.

The restaurant prefers the DF contract, except for the extremely high supply costs. This contract's advantage for the restaurant lies in its ability to align better the incentives of both the platform and the restaurant to lower their margins and boost online orders. Under this contract, the platform commits to an ample supply of drivers first through high wages. The restaurant would then reduce its margin, knowing that the platform does not have any incentives to charge a high delivery fee for online channel customers as it is already committed to an ample labor supply of drivers. Such coordination in the online channel pricing benefits the restaurant and maximizes the whole chain's profit among all these contracting schemes (unless the supply cost is high). Given the platform's preference for the FD contract and the DF contract's optimality for the restaurant and the overall food delivery chain, we propose a new DF contract with subscription fees. This revised contract could enhance profits for both the platform and the restaurant and improve the surplus for drivers and customers compared to the platform's preferred FD contract.

With the rise of the sharing economy, the employment status of gig workers has become a contentious issue globally (Sun et al. 2023). Regulators are considering setting either a minimum wage (per hour) or a wage rate (per delivery) to enhance drivers' welfare. Our analysis reveals that these approaches have different implications for the food delivery market. While a relatively high minimum wage can help the food delivery chain and all parties involved in the online food delivery market benefit (only the platform loses), a relatively high wage rate can harm all players in the food delivery market. In particular, we show that the platform's commitment to a low enough wage rate can improve the coordination between the restaurant and the platform's pricing, resulting in competitive online prices. This increased competition can drive up online orders, benefiting drivers, customers, the restaurant, and the overall food delivery chain.

The rest of the paper is organized as follows. First, we review the literature in Section 2; then, we introduce our model in Section 3. The following section elaborates on the analysis, while Section 5 compares the performances of different contracting schemes. Section 6 presents numerical experiments that help us improve our understanding of the problem, while Section 7 incorporates two extensions of the model. Finally, we conclude with managerial insights in Section 8.

2. Related Literature

This paper contributes to the literature on multi-sided markets. Amid the extensive literature on two-sided markets in economics (e.g., Caillaud and Jullien 2003, Rochet and Tirole 2003, Andrei 2009), a growing body of operations management literature has studied the sharing economy. Within this field, some papers focus on the operational aspects of price and wage design. For instance, Banerjee et al. (2016) examine a scenario where the wage is an exogenous proportion of the price to show that static pricing is effective. In contrast, Cachon et al. (2017) study pricing schemes in which both the price and wage are endogenous. They find that surge pricing can achieve nearly optimal profit, and all stakeholders can benefit from surge pricing on a platform with self-scheduling capacity. Hu and Zhou (2020) show that it is optimal for the platform to offer a fixed ratio commission for drivers, which depends on the price and wage sensitivity coefficients of the linear demand and supply functions, while Garg and Nazerzadeh (2022) propose an incentive-compatible pricing mechanism for drivers in response to surge pricing. Taylor (2018) examines how delay sensitivity and agent independence affect a platform's endogenous pricing and waging decisions. Our context differs from the above literature, as it investigates the contractual relationships not only between the platform and drivers but also between the platform and the restaurant.

In two-sided markets, researchers have studied precommitment to wage or price in competitive settings. Hu and Liu (2023) investigate how commitment to price or wages can soften market competition. In particular, they show that platforms can benefit from commitment through softened competition if competing platforms commit to wages in the supply market or prices in the demand market, whichever is less competitive. In particular, this study extends the Kreps and Scheinkman equivalency (Kreps and Scheinkman 1983), demonstrating that precommitment to capacity results in reduced price competition. Zhang et al. (2022a) examine three common contracting schemes in two-sided markets, investigating the role of self-scheduling drivers on the platform's profitability. They demonstrate how the relative intensity of competition in the demand vs. supply market affects the platform's choice of contract. Moreover, they reconfirm the findings of Hu and Liu (2023). In contrast to these papers, while investigating wage commitment, we consider a three-sided market where market competition is between a platform's online channel and a restaurant's dine-in offline channel.

This research mainly contributes to online food delivery (OFD) services literature. Within this field, one stream of studies investigates factors that affect delivery performance. For example, Mao et al. (2019) empirically show that a driver's individual local area knowledge and prior delivery experience can reduce late deliveries significantly. Based on data from a major Chinese food delivery platform, Zhang et al. (2023) show a high restaurant density reduces the delivery speed. Additionally, Chen and Hu (2024) examine the impact of dedicated versus pooling dispatch strategies on delivery performance, mainly when customers are sensitive to delays. Several studies empirically examine the economic effects of on-demand delivery platforms on restaurants. For example, Li and Wang (2024a) show that, generally, restaurants can benefit from selling through delivery platforms, and the overall positive effect on fast food chains is stronger than that on independent restaurants. Unlike the above findings, Karamshetty et al. (2023) illustrate that the dependence on the platform might reduce the sales revenue from high-margin items.

Another stream of research in this area focuses on the contract design between platforms and restaurants to achieve better performance for the food delivery chain members. Specifically, Oh et al. (2023) show that a contract with sharing food revenue and splitting the delivery costs and fees between platforms and restaurants can achieve the first-best profits. Similar to this paper, Feldman et al. (2023) and Chen et al. (2022) also illustrate that simple revenue sharing has inherent drawbacks and fails to coordinate the system. In Feldman et al. (2023), they propose a generalized revenue-sharing contract that can coordinate the system, while Chen et al. (2022) find that a simple revenue-sharing contract with a “price ceiling” on the delivery menu price coordinates the system. Regarding the regulatory issue in the context of on-demand food delivery, Li and Wang (2024b) discusses whether the government should set an upper bound on the commission rates asked by platforms. Zhang et al. (2022b) investigate the government’s policy design to curb traffic incidents brought by delivery drivers. While these studies enhance our understanding of the online food delivery market from various perspectives, they often focus on issues involving only one or two parties, neglecting the market’s three-sided nature. In contrast, our research models the contractual relationships among platforms, restaurants, and drivers to capture the intricate dynamics of this market.

A few recent studies on online food delivery platforms consider the market’s three-sided nature. Bahrami et al. (2023), for instance, characterize the optimal commissions and wages from the perspective of a profit-maximizing or welfare-maximizing platform when customers are time-sensitive. Liu et al. (2023) adopt a state-dependent queuing model to study the platform’s revenue maximization problem, where customers, deliverers, and restaurants make independent participation decisions. Unlike these studies, we consider the sequential moves in contracting in the three-sided market and focus on contractual performance. In this line of research, Sun et al. (2023) are perhaps the closest to our work. Sun et al. (2023) study the three-sidedness of the OFD market and examine two competing platforms’ optimal choices in a setting where the platforms compete on both prices and service quality. They show that the platforms’ incentive to exploit the market’s three-sided nature is significantly affected by two key factors: whether consumers benefit from service improvement and the intensity of interaction in the buyer-seller market. Our paper differs from this paper in several distinct ways. First, their study considers the competition between platforms, whereas we consider a single platform in the market and we examine competition between the online and offline channels. Moreover, the emphasis in our paper is on the contractual relationships between a platform, a restaurant, and self-scheduling drivers.

3. Model

We consider a stylized three-sided food delivery market in which an online platform connects a restaurant, a group of delivery drivers, and customers seeking catering services. The restaurant contracts with the platform to expand its market base so customers can order food online through the platform. Since delivery drivers are self-scheduling and have alternative work options, the platform must offer competitive wages to incentivize them to fulfill online orders. Besides the online channel, the restaurant also provides a dine-in offline channel, where customers can commute to the restaurant and get dining service.

3.1. Demand Specification

We assume the online and offline channels are differentiated, and customers are sensitive to prices in these channels. We use a linear demand system to model the channel demands as follows.

$$q_o = \tfrac{1}{1+\beta} - \tfrac{1}{1-\beta^2} p_o + \tfrac{\beta}{1-\beta^2} p_f, \tag{1}$$
$$q_f = \tfrac{1}{1+\beta} - \tfrac{1}{1-\beta^2} p_f + \tfrac{\beta}{1-\beta^2} p_o, \tag{2}$$

where q_o is the online demand, i.e., orders placed at the platform, and q_f represents the dine-in offline demand. p_o and p_f represent full market prices in these channels, including delivery fees or discounts for online and offline prices. The marketing and economics literature has extensively studied such linear demand systems to model differentiated duopolies (Singh and Vives 1984, Jerath and Zhang 2010). In the above model, β ($0 \leq \beta \leq 1$) represents the degree of differentiation between the online and offline channels. When β equals zero, the two channels operate independently. As β increases, the competition between these two channels becomes more intense. When β approaches one, the two channels become fully substitutable, leading to perfect competition between the two channels in the market. Intermediate values of β represent varying degrees of differentiation.

We use this demand specification because it has two desirable characteristics: as the differentiation between these two channels increases (i.e., β decreases), the price sensitivity $1/(1-\beta^2)$ decreases. This is consistent with the idea that customers are less price-sensitive to more differentiated products. Additionally, the total potential market size in this demand model $2/(1+\beta)$ decreases in β, which aligns with the intuition that more differentiated products can reach a broader customer base. We have also explored other popular demand models where the total market size remains unaffected by the degree of product differentiation. In the extensions, we evaluate how variations in the relative market potentials for online and offline markets impact our findings.

3.2. Labor Supply of the Delivery Drivers

The platform must attract enough drivers to provide delivery services for customers ordering at the online channel. We use a linear model to characterize the labor supply of the delivery drivers, which increases with the wage offered by the platform. In particular, we assume that the labor supply of the delivery drivers is given by,

$$s(w) = [-a + bw]^+, \tag{3}$$

where w represents the wage paid by the platform to the delivery drivers, and $s(w)$ denotes the labor supply provided by the delivery drivers. a ($a \geq 0$) represents the attraction of working options other than the online platform for drivers, and b measures the delivery drivers' sensitivity to the platform's wage. A lower value of a indicates that working for the platform is becoming more attractive compared to the outside option (for example, due to the flexibility of the self-scheduling feature of the platform). In contrast, a lower b indicates that working for the platform vs. the outside options is becoming less of a substitute, i.e., attracting drivers becomes more costly. In addition to the above wage-dependent model, an alternative formulation based on the wage rates is developed in Section 7.

A transaction in the online channel happens only when the platform matches an online order with a delivery driver. Suppose the online demand exceeds the labor supply of the drivers. The platform will randomly assign a limited supply of drivers to the online orders, and the food delivery chain will lose the unsatisfied demand. If the labor supply exceeds the online demand, the platform will assign limited online orders to the drivers (randomly), and the extra supply will be wasted. Therefore, the transaction volume of the online channel in our model is given by $\min(s(w), q_o)$. This proportional rationing is a quite common assumption in the literature (e.g., Hu and Liu 2023, Zhang et al. 2022a).

3.3. The Platform and the Restaurant

The platform and the restaurant cooperate in the online channel to provide food delivery services to customers; at the same time, the online channel competes with the offline dine-in channel that the restaurant completely controls. Both firms seek to maximize their profits. We can formulate the profit function for the platform as

$$\pi_p^i = \min(s(w), q_o)(m_p^i - w), \tag{4}$$

and the profit function for the restaurant as

$$\pi_r^i = \min(s(w), q_o)m_r + p_f q_f, \tag{5}$$

where the superscript $i \in \{FD, DD, DF\}$ represents three different contracting schemes between the platform and the restaurant, as we will introduce in detail in the next subsection. m_p^i and m_r

are the online channel profit margins for the platform and the restaurant, respectively. Unlike m_r, m_p^i can take different forms in different contracting schemes, as we will show in detail in Section 4. We simplify m_r to m throughout the paper.

3.4. Contracting Schemes

We study the following three contracting schemes governing the relationships between the restaurant, the platform, and delivery drivers in this three-sided food delivery market.

Fixed-price/dynamic-wage (FD) contract Under this contract, the platform sets a commission fee charged to the restaurant first, allowing the restaurant to set the online channel price (that might include a delivery fee) alongside the offline channel price next. The platform then moves last to set the wage for the delivery drivers. The FD contract indicates that when the platform sets its wages for the delivery drivers, the online channel price is already known and set by the restaurant, i.e., the labor supply side of the platform is set after the demand side.

Dynamic-price/dynamic-wage (DD) contract Under this contract, the platform first asks for a commission fee per delivery. Next, the restaurant sets its margin for online orders alongside the dine-in offline prices. Then, the platform sets the final online price by posting its delivery fee or even providing a discount to online customers. Meanwhile, it also sets the wage for the delivery drivers. The DD contract refers to the fact that both the online channel price and the drivers' wage are set simultaneously in the market, i.e., the labor supply and demand sides are set simultaneously.

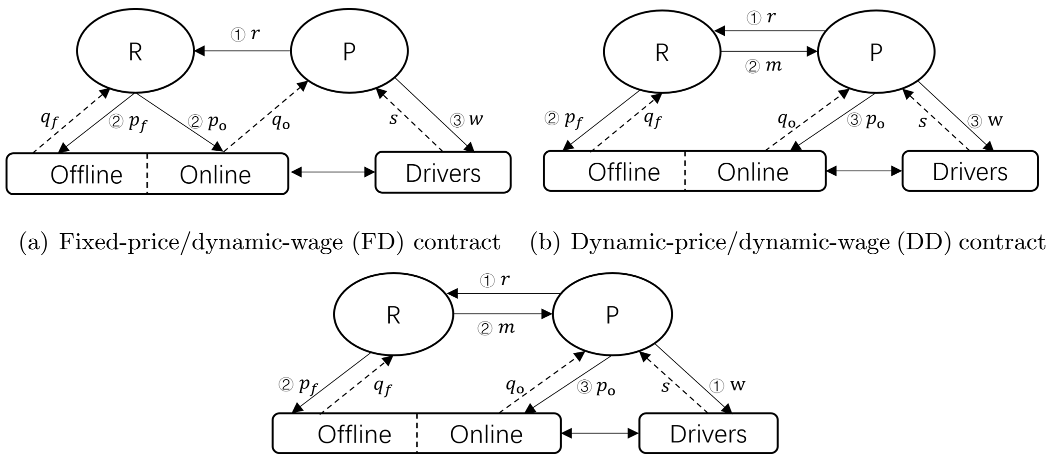
Dynamic-price/fixed-wage (DF) contract Under this contract, the platform first announces the wage for the delivery drivers alongside the commission fee per delivery to the restaurant. The restaurant then sets the margin it charges the platform for online orders alongside the dine-in offline prices, and the platform announces its delivery fees or discounts to set the online channel price at the end. The DF contract refers to the fact that when the platform sets its online channel price in the market, the wage for the drivers is already known as the platform has committed to it at the first stage, i.e., the labor supply side of the platform is set before the demand side.

Figure 1 provides a schematic view and decision sequence of the three contracting schemes. To rule out trivial cases, we make the following assumption throughout this study.

Assumption 1. *The model parameters satisfy* $b \geq \frac{a}{1-\beta}$.

This assumption implies that the delivery drivers are responsive enough to the wages, so the cost of providing incentives for delivery drivers is not excessively high as a low b might imply. This assumption ensures that the demands of the online and offline channels are positive in all three contract schemes. If not, the platform and the restaurant will find the online channel unprofitable. We use backward induction to establish the equilibrium contracts. We summarize the notations used in this paper in Table A.1, and all proofs are provided thereafter.

Figure 1: Schematic View of the Three-Sided Market under FD, DD, and DF Contracts.



(a) Fixed-price/dynamic-wage (FD) contract (b) Dynamic-price/dynamic-wage (DD) contract

(c) Dynamic-price/fixed-wage (DF) contract

3.5. Benchmark Models

In this subsection, we study two benchmark models: The centralized case and the fixed-labor-supply case. First, in the centralized case, we assume that a decision maker sets the online and offline channel prices and the drivers' wages to maximize the profit of the entire food delivery chain. It allows us to compare the performance of different contracting schemes vs. that of the best achievable profit. Second, in the fixed-labor-supply case, we assume that the labor supply of the drivers is exogenously given. It allows us to understand better the effect of the self-scheduling labor supply in the sharing economy. In what follows, we first characterize the equilibrium decisions of the players in the two benchmark models. Then, we investigate the equilibrium outcomes of the three contracting schemes in the sharing economy in the next section.

3.5.1. The Centralized Case In this benchmark case, a centralized decision maker sets the online and offline channel prices, p_o and p_f, and the wages for the delivery drivers w to maximize the profit of the food delivery supply chain:

$$\Pi^C = \max_{w,p_o,p_f} \min(s(w), q_o)(p_o - w) + p_f q_f, \tag{6}$$

where Π^C is the sum of the sales profit from the online and offline channels. This case serves as a benchmark to evaluate the performance of different contracting schemes for the entire food delivery chain. The following lemma characterizes the equilibrium outcomes (denoted by superscript "C").

LEMMA 1. *In the centralized model, the online and offline channel prices and the wages for the drivers are,*

$$p_o^{C*} = \tfrac{(1-\beta^2)(a+b)+2-\beta}{2b(1-\beta^2)+2}, \tag{7}$$

$$p_f^{C*} = \tfrac{1}{2}, \tag{8}$$

$$w^{C*} = \tfrac{a+b(1-\beta)+2ab(1-\beta^2)}{2b(1+b(1-\beta^2))}. \tag{9}$$

3.5.2. The Fixed-Labor-Supply Case In this benchmark case, we study three sub-cases of the contracting schemes introduced in the main model but assume that the platform has a fixed supply of drivers, denoted as $\hat{s}$, and their wage is also constant at c. The profit functions of the platform and the restaurant are,

$$\pi_p^j = \min(\hat{s}, q_o)(m_p^j - c), \tag{10}$$
$$\pi_r^j = \min(\hat{s}, q_o)m + p_f q_f, \tag{11}$$

respectively, where $j \in \{BFD, BDD, BDF\}$, represents the benchmark sub-cases of three contracting schemes (the FD, DD, and DF contracts) with fixed labor supply of drivers (see Section C.2 in Online Appendix C for more details). Comparing these benchmark models with the ones with self-scheduling drivers can help us evaluate the impact of the sharing economy. The following lemma establishes the equilibrium outcomes of these benchmark sub-cases.

Lemma 2. *If the labor supply of the drivers is given by $\hat{s}$, we can establish the following.*

(i) The equilibrium online and offline channel prices of the three benchmark sub-cases are the same as follows,

$$(p_o^{BFD*}, p_f^{BFD*}) = (p_o^{BDD*}, p_f^{BDD*}) = (p_o^{BDF*}, p_f^{BDF*}) = \begin{cases} (\frac{2-\beta-2\hat{s}(1-\beta^2)}{2}, \frac{1}{2}) & if \quad \hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}, & (12a) \\ (\frac{(3-\beta+c)}{4}, \frac{1}{2}) & if \quad \hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}. & (12b) \end{cases}$$

(ii) The equilibrium online demands of the three benchmark sub-cases are also the same as,

$$q_o^{BFD*} = q_o^{BDD*} = q_o^{BDF*} = \begin{cases} \hat{s} & if \quad \hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}, & (13a) \\ \frac{1-\beta-c}{4(1-\beta^2)} & if \quad \hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}. & (13b) \end{cases}$$

Lemma 2 shows that when the exogenously given labor supply is ample, the platform and the restaurant choose the profit-maximizing prices; otherwise, the online channel price is a function of labor supply, and the fixed size of supply limits the online sales. Additionally, an important observation in the above lemma is that these three contracting schemes result in the same online/offline channel prices and demands when the labor supply of delivery drivers is exogenous. Consequently, the identical market outcomes lead to the same food delivery chain's profit across all contracts. Following Lemma 2, we express the contract equivalency for benchmark cases next.

Corollary 1. *If the labor supply of drivers is exogenous, then the FD, DD, and DF contracts result in the same market outcomes.*

This corollary implies that when the supply is exogenously given, the contracting dynamics between the platform and the restaurant do not affect the market outcomes; the change in the decision sequences in these contracts only affects the profit distribution between the firms. In the next section, we demonstrate that the established equivalency in Corollary 1 fails to hold as we incorporate the sharing economy with self-scheduling delivery drivers.

4. Analysis

In this section, we first derive the equilibrium outcome in the sharing economy, where the drivers are self-scheduling, responding to changes in the platform's waging under different contracting schemes. Then, we compare the market outcomes of the three contracting schemes with the benchmark cases to show the impact of self-scheduling delivery drivers. Finally, we present a sensitivity analysis of the market outcomes concerning the channel differentiation and labor supply parameters.

4.1. The Fixed-Price/Dynamic-Wage (FD) Contract

Under the FD contract, the platform first announces the commission fee r it charges to the restaurant. Then, the restaurant decides the offline channel price alongside the online price by setting the online channel profit margin m before the platform finally sets up the wage offered to the delivery drivers. Therefore, the margin for the platform is given by $m_p^{FD} = r$, and it chooses a commission fee r to maximize its profit in the first stage:

$$\max_{r} \pi_p = \min(s(w), q_o)(r - w). \tag{14}$$

Given the commission fee, the restaurant sets the offline channel price p_f and the online channel profit margin m to maximize the following profit function in the second stage,

$$\max_{m,p_f} \pi_r = \min(s(w), q_o)m + q_f p_f. \tag{15}$$

The online channel price charged to the customers is composed of the commission fee charged by the platform and the margin charged by the restaurant, which is given by $p_o = r + m$ under this contracting scheme. Therefore, the online and offline channel demands q_o and q_f are realized. Finally, in the last stage of the game, given the commission fee and the online and offline prices, the platform sets the wage w for the drivers to provide the labor supply $s(w)$ to maximize the following,

$$\max_{w} \pi_p = \min(s(w), q_o)(r - w). \tag{16}$$

Solving backward, we can characterize the equilibrium outcomes in the following lemma.

LEMMA 3. *Under the FD contract, in equilibrium, the commission fee, the online and offline channel prices, and the wage for the drivers are*

$$r^{FD*} = \tfrac{(1-\beta)(1+a(\beta+1)+b(1-\beta^2))}{1+2b(1-\beta^2)}, \tag{17}$$

$$p_o^{FD*} = \tfrac{1}{2} + \tfrac{(1-\beta)(1+a(\beta+1)+b(1-\beta^2))}{2+4b(1-\beta^2)}, \tag{18}$$

$$p_f^{FD*} = \tfrac{1}{2}, \tag{19}$$

$$w^{FD*} = \tfrac{a+4ab(1-\beta^2)+b(1-\beta)}{4b^2(2-\beta^2)+2b}. \tag{20}$$

Substituting for the equilibrium decisions, we can show that the platform matches supply and demand. If the labor supply of the drivers exceeds the online channel demand, the transaction volume matches the online demand. The platform can reduce the wage for the drivers to maintain the same transaction volume but result in a higher profit margin for the online channel. Therefore, in equilibrium, the labor supply cannot exceed the online demand. On the other hand, if the labor supply is smaller than the online channel demand, the transaction volume matches the labor supply. Given the fixed commission fee, the platform's profit depends only on the supply side and is unrelated to the demand side. Therefore, the restaurant can increase the online channel price to increase its profit margin until the online demand matches the supply.

4.2. The Dynamic-Price/Dynamic-Wage (DD) Contract

Under the DD contract, the platform first announces the commission fee charged to the restaurant. Then, the restaurant sets the dine-in offline channel price alongside the margin it charges the platform for online sales. Finally, the platform sets the delivery fee of d to adjust the online channel price alongside the delivery drivers' wages. Therefore, the online channel profit margin for the platform is $m_p^{DD} = r + d$, and it chooses a commission fee r to maximize its profit at the first stage:

$$\max_{r} \pi_p = \min(s(w), q_o)(r + d - w). \tag{21}$$

In the second stage, the restaurant decides the offline channel price alongside the online sales margin m to maximize its profit, which is given by

$$\max_{m,p_f} \pi_r = \min(s(w), q_o)m + q_f p_f. \tag{22}$$

Finally, in the last stage of the game, the platform sets the delivery fee for the online customers and the wage for the drivers to maximize its profit:

$$\max_{w,d} \pi_p = \min(s(w), q_o)(r + d - w). \tag{23}$$

The online channel price charged to the customers comprises the commission fee and delivery fee charged by the platform and the margin charged by the restaurant, which is given by $p_o = r + m + d$. The online and offline demands are then realized after the platform sets the delivery fee. We employ backward induction to derive the equilibrium outcomes under the DD contract.

Lemma 4. *Under the DD contract, in equilibrium, the online and offline channel prices and the wage for the drivers are given by,*

$$p_o^{DD*} = \frac{(1-\beta^2)(a+b(3-\beta))+2(2-\beta)}{4b(1-\beta^2)+4}, \tag{24}$$

$$p_f^{DD*} = \frac{1}{2}, \tag{25}$$

$$w^{DD*} = \frac{a(3+4b(1-\beta^2))+b(1-\beta)}{4b(b(1-\beta^2)+1)}. \tag{26}$$

The first observation in the above lemma is that the labor supply matches the online demand. Unlike the FD contract, the platform under the DD contract utilizes both the delivery fee and the wage for drivers to match the labor supply and the demand for the online channel. Intuitively, if the online demand exceeds the labor supply on the platform, the transaction volume will equal the number of drivers available. In this case, if the wage remains constant and the platform slightly increases the delivery fee, it can maintain the same transaction volume while achieving a higher profit margin. Therefore, demand cannot exceed supply in equilibrium. Similarly, supply cannot exceed the online demand in equilibrium because the platform could increase its profit by reducing the wage without lowering the transaction volume.

The next observation indicates that the commission fee set by the platform does not affect the equilibrium outcome. The proof section demonstrates that the restaurant margin for online sales is independent of r, and the online channel price set by the platform (that includes the delivery fee) serves as a perfect substitute for the commission fee (see equation (C.24) in Online Appendix C). This means that any increase in the commission fee charged by the platform would lead to a decrease in the online channel price, while a reduction in the commission fee would result in a higher online channel price. The Doordash marketplace falls into this category of contracts. It offers Basic, Plus, and Premier Partnership Plans to restaurants, each with progressively higher commission fees. Customers pay lower service and delivery fees to the platform when the restaurant subscribes to the Plus and Premier Partnership Plans because these delivery fees decrease in the commission fees paid by restaurants (Doordash 2023a).

4.3. The Dynamic-Price/Fixed-Wage (DF) Contract

Under the DF contract, the platform first commits to the wage paid to the delivery drivers and asks for a commission from the restaurant. The restaurant then sets the dine-in offline channel price alongside the margin charged to the platform for online orders. Finally, the platform sets the online channel price by announcing the delivery fee for the customers. Therefore, the online channel profit margin for the platform is $m_p^{DF} = r + d$ under this contract. We can write the platform's problem in the first stage of the game as a function of the wage w and commission fee r,

$$\max_{w,r} \pi_p = \min(s(w), q_o)(r + d - w). \tag{27}$$

In the second stage, the restaurant sets the margin on online orders m alongside the dine-in offline channel price p_f to maximize its profit, which is given by

$$\max_{m,p_f} \pi_r = \min(s(w), q_o)m + q_f p_f. \tag{28}$$

Finally, the platform sets the online channel price by announcing the delivery fee d in the third stage to maximize its profit:

$$\max_{d} \pi_p = \min(s(w), q_o)(r + d - w). \tag{29}$$

Similar to the DD contract case, the online and offline demands are realized after the platform sets the delivery fee, where the online channel price is given by $p_o = r + d + m$. We solve for the equilibrium outcomes by backward induction, characterized by the following lemma.

Lemma 5. *Under the DF contract, in equilibrium, the online and offline channel prices and the wages for the drivers are*

$$p_o^{DF*} = \frac{2a(1-\beta^2)+2b(3-\beta)(1-\beta^2)+2-\beta}{8b(1-\beta^2)+2}, \tag{30}$$
$$p_f^{DF*} = \frac{1}{2}, \tag{31}$$
$$w^{DF*} = \frac{1-\beta+4a(1-\beta^2)}{1+4b(1-\beta^2)}. \tag{32}$$

Like the previous two contracting schemes, the online channel demand matches the labor supply in equilibrium under the DF contract. The platform sets the wage for the drivers before competing with the restaurant's offline channel. Once the wage is given, the number of drivers working for the platform (supply capacity) is fixed. If the realized online orders exceed the fixed supply, then the transaction volume is given by the supply of the drivers. Therefore, the restaurant and platform can increase profit margins without changing the transaction volume. Conversely, suppose the labor supply exceeds the online demand. In that case, the platform can decrease delivery drivers' wages in the first stage without changing the transaction volumes and the restaurant's pricing decisions.

An interesting observation about all these contracting schemes is given in the following corollary.

Corollary 2. *The equilibrium dine-in offline channel price is independent of the contracting scheme between the restaurant, platform, and delivery drivers and is equal to those of the centralized and fixed-labor-supply benchmark cases.*

The equilibrium outcomes under all these contracting schemes indicate that the restaurant always prefers to set the dine-in offline channel price equal to $\frac{1}{2}$, independent of the contracting scheme. This observation corroborates that restaurants do not frequently change their dine-in prices, while they might change their online prices more regularly.

4.4. The Impact of Self-Scheduling Drivers

To examine the impact of the self-scheduling drivers, we compare the equilibrium outcomes of the three contracts in the fixed-labor-supply case (the second benchmark case, see Lemma 2 in §3.5.2) with those under the sharing economy, where the labor supply of the drivers is self-scheduled. In the fixed-labor-supply case, we assume a fixed number of drivers $\hat{s}$ working for the platform, and the wage for the drivers c is exogenous. In the sharing economy, the drivers are self-scheduling, and the platform can adjust the labor supply of drivers through the wage. To make a "fair" comparison, we assign the value of c in the fixed-labor-supply models to the equilibrium wage value of the three contracting schemes, respectively. The following proposition characterizes our findings.

Proposition 1. *The equilibrium online channel price is lower in the sharing economy case than that of the fixed-labor-supply case if and only if the equilibrium driver supply in the sharing economy is greater than the fixed-labor-supply drivers $\hat{s}$.*

Proposition 1 indicates that the sharing economy with self-scheduling drivers might intensify or soften market competition compared to the fixed-labor-supply case, dependent on the level of the fixed labor supply but regardless of the contracting schemes. In particular, if the fixed supply of the drivers is limited, the platform and the restaurant will jointly set a high online channel price to ensure a large profit margin. In contrast, in the sharing economy, the platform and the restaurant will find it profitable to serve more customers by lowering the price in the online channel and adjusting the delivery drivers' wages. If the fixed labor supply is ample, the dynamics between the platform and the restaurant will lead to fierce competition between the online and offline channels. However, the platform and the restaurant would soften the competition in the sharing economy by adjusting the delivery drivers' wages and online channel prices.

The above finding is similar to the one discussed in Zhang et al. (2022a) in a two-sided market, where two platforms compete for drivers and customers. They show that the effect of the sharing economy on market competition is a function of the exogenous supply of drivers for these platforms. If the fixed supply of the competing platforms exceeds the equilibrium supply in the sharing economy, adopting the sharing economy would soften market competition between these platforms. We extend their findings to a three-sided market, where a platform interacts with drivers and a restaurant to provide food delivery services to online customers, competing with the dine-in offline channel. If the platform has an ample fixed supply of drivers, diverting to a sharing economy model to supply drivers can help the platform soften price competition in the market. However, as we will demonstrate later, it might not be optimal for the food delivery chain to soften the competition between the online and offline channels.

In the fixed-labor-supply case, the market outcomes are independent of the contractual structure (see Corollary 1). However, this independence does not hold in the sharing economy, where the platform can leverage the wage to control the labor supply of drivers.

Proposition 2. *The equilibrium market outcomes in the sharing economy depend on the contract schemes. In particular,*

(i) The DD contract results in the highest, and the DF contract in the lowest online channel prices, i.e., $p_o^{DD*} \geq p_o^{FD*} \geq p_o^{DF*}$.

(ii) The DF contract generates the highest, and the DD contract the lowest online demands, i.e., $q_o^{DF*} \geq q_o^{FD*} \geq q_o^{DD*}$.

(iii) The DF contract offers the highest, and the DD contract the lowest wages for the drivers, i.e., $w^{DF*} \geq w^{FD*} \geq w^{DD*}$.

Part (i) of Proposition 2 shows that the market competition would be softened in the sharing economy if the platform determines the final online channel price, setting the delivery fees, and the delivery driver's wages simultaneously. In the DD contract, the platform has two levers to match drivers' supply with online channel demand in the market, i.e., changing the final online channel price by charging different delivery fees or customizing the wages it offers to the delivery drivers. Such flexibility, on the platform side, benefits the restaurant by charging a higher margin than the other contracting schemes, in which the platform has only one lever to match supply and demand, i.e., only the wage or the final online price.

Under the FD contract, the restaurant sets the online channel price while the platform controls the delivery drivers' wages to manage the labor supply. Since the only lever the platform can use to match the labor supply with the online demand in the market is the delivery wage offered to the drivers, the restaurant has to reduce its margin to induce larger online orders, incentivizing the platform to raise the drivers' supply through higher wages. This results in a more competitive pricing strategy for the restaurant and generally higher wages compared to the DD contract.

Proposition 2 also indicates the most fierce competition in the food market happens under the DF contract. Under this contract, the platform can set the labor supply of the drivers before getting involved in the competition with the restaurant's dine-in offline channel. We show that the platform should provide a large supply of drivers when competing with the dine-in offline channel. After the platform's commitment to an ample supply of drivers (compared with the other contracting schemes), the restaurant expects low delivery fees in the market as the platform has already committed to an ample supply of drivers. Therefore, the restaurant reduces the margin charged to the platform to benefit from larger online orders. In other words, with a commitment to an ample supply, both firms can coordinate to reduce their margins in the online channel, increasing the online channel's competitiveness.

Our findings in part (i) justify parts (ii) and (iii), given that Corollary 2 establishes that the offline prices are the same under all contracting schemes. Therefore, a lower online price indicates a higher online demand, which also requires a higher wage to match the online demand and the labor supply of the drivers.

The above findings deviate from the literature on quantity-then-price competition. The literature suggests that when firms initially compete based on quantities and then on market prices, they tend to limit their capacities to mitigate price competition later in the market (Kreps and Scheinkman 1983). However, this differs from our observations under the DF contract. While the platform's announced wage in the first stage indicates a capacity commitment, it is optimal for the platform to commit to an ample supply (Part (ii) of Proposition 2) to intensify the market competition between the online and offline channels. The DF contract fundamentally differs from the capacity-then-price competition model in the literature. In the DF contract, the platform and the restaurant move sequentially. After the platform's commitment to the labor supply of the drivers, the restaurant moves next to set its margin per unit sold in the online channel alongside the dine-in offline prices. Moreover, we assume the restaurant has an unlimited capacity (as it does not need delivery drivers) to serve the dine-in customers. Given these distinct features of the food delivery market, we show that the platform should commit to an ample supply of drivers to induce the restaurant to reduce its margin and make the online channel more competitive. In other words, if the platform commits to a low capacity to curb market competition in the first stage (similar to the capacity-then-price competition), it is the restaurant that would benefit from the softened competition by charging a high margin as the restaurant moves next, which hurts the platform's profitability.

4.5. Sensitivity Analysis

In this subsection, we explore the impact of the substitutability of the online and offline channels β on the market equilibrium under different contracting schemes. Additionally, we investigate how the characteristics of the drivers' supply market, i.e., a and b, affect the equilibrium outcomes, where a represents the attraction of outside options other than the online platform for the drivers and b measures the supply side sensitivity to wages. We establish the marginal impact of these parameters in the following proposition.

Proposition 3. *Under the contracting scheme $i \in \{FD, DD, DF\}$, we can establish:*

(i) The online channel price p_o^{i*} increases in a and decreases in b and β.

(ii) The online channel demand q_o^{i*} decreases in a and β and increases in b.

(iii) The wage w^{i*} increases in a and decreases in b and β.

(iv) The platform's and restaurant's profits decrease in a and β and increase in b.

The first observation in Proposition 3 is that the effect of different parameters on the equilibrium outcomes is quite similar under all contracting schemes. Part (i) shows that as the channel substitutability increases (i.e., as β increases), the online channel price decreases (note that the offline channel price stays the same). As the online and offline channels become more substitutes, the platform (or the restaurant under the FD contract) should decrease its online prices. Similarly, the driver's supply becomes less costly as b increases or a decreases. A cheaper supply indicates that the online channel price can be more competitive than the offline price.

Part (ii) indicates that the online demand decreases as the online and offline channels become more substitutable. To understand why, we should recall our assumption about the characteristics of the demand model. For many markets, including our online food delivery market, it is reasonable to assume that less differentiated products will reach a smaller market (Abhishek et al. 2016). As the online and offline channels become less differentiated, the total market potential for the food delivery chain decreases. The platform (or the restaurant) has to reduce online food prices to fight back. While the concession in online prices can alleviate the effect of an increase in β, the online demand still suffers and would decrease. Such a reduction in online demand also indicates that the wages for the delivery drivers would decrease in β.

Our observation in part (ii) also justifies the findings on the delivery drivers' wages. In particular, as the competition between the online and offline channels increases, the reduction in the online demand and the fact that the platform matches supply with demand, in equilibrium, indicates that the required wage for delivery drivers would decrease.

The above proposition also shows that both the platform and the restaurant would lose as the online and offline channels become less differentiated or as the supply market becomes more expensive (i.e., an increase in a or a decrease in b). A less differentiated food delivery market indicates lower online demands with lower online channel prices. While it also indicates lower wages for drivers, the reduction in online demand has a more profound effect on the profitability of both the platform and the restaurant. Similarly, a more expensive supply market cannot help any of these firms benefit as they need to share the burden of a more expensive supply.

5. Comparison of Contracting Schemes

The online food delivery market features a variety of contracting schemes, prompting a key question for all involved parties: the platform, the restaurant, the customers, and the delivery drivers. Which contracting scheme is the most advantageous from each one's perspective? This question gains significance considering the earlier findings that the equilibrium outcomes of the three contracting schemes may display different characteristics. Moreover, this question is relevant in the sharing economy, as all contracting schemes result in the same market outcomes in the traditional economy where the labor supply of the drivers is fixed. In the sharing economy, the platform has to provide the right incentive to the delivery drivers to match their labor supply with the online demand.

5.1. The Platform's and the Restaurant's Preference over the Contracting Schemes

The following proposition compares the platform's and the restaurant's equilibrium profits under different contracting schemes. Moreover, we present our findings for the food delivery chain's profit, which is defined as the sum of the platform and the restaurant's profit, i.e., $\pi_{sc}^{i*} = \pi_p^{i*} + \pi_r^{i*}$.

PROPOSITION 4. *We can establish the following:*

(i) For the platform, if $b \geq \frac{1}{8(1-\beta^2)}$*, then* $\pi_p^{FD*} \geq \pi_p^{DF*} \geq \pi_p^{DD*}$*; otherwise,* $\pi_p^{FD*} \geq \pi_p^{DD*} \geq \pi_p^{DF*}$*.*

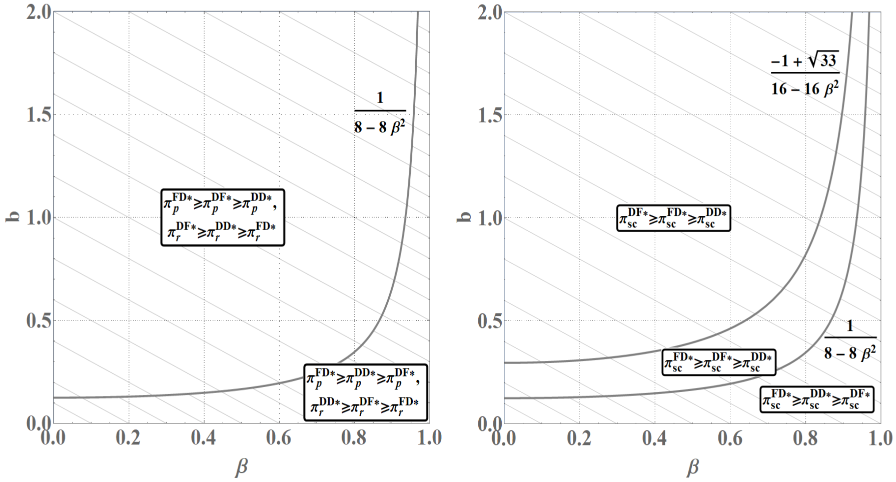
(ii) For the restaurant, if $b \geq \frac{1}{8(1-\beta^2)}$*, then* $\pi_r^{DF*} \geq \pi_r^{DD*} \geq \pi_r^{FD*}$*; otherwise,* $\pi_r^{DD*} \geq \pi_r^{DF*} \geq \pi_r^{FD*}$*.*

(iii) For the food delivery chain, if $b \geq \frac{-1+\sqrt{33}}{16(1-\beta^2)}$*, then* $\pi_{sc}^{DF*} \geq \pi_{sc}^{FD*} \geq \pi_{sc}^{DD*}$*; if* $\frac{1}{8(1-\beta^2)} \leq b \leq \frac{-1+\sqrt{33}}{16(1-\beta^2)}$*, then* $\pi_{sc}^{FD*} \geq \pi_{sc}^{DF*} \geq \pi_{sc}^{DD*}$*; otherwise,* $\pi_{sc}^{FD*} \geq \pi_{sc}^{DD*} \geq \pi_{sc}^{DF*}$*.*

Part (i) of Proposition 4 indicates that the platform always favors the FD contract, while the worst performance is attributed to the DD/DF contracts. Specifically, when the cost of supply is excessively high (i.e., $b \leq \frac{1}{8(1-\beta^2)}$, please refer to Figure 2), the performance of the DD contract for the platform is better than the DF contract; otherwise, the platform prefers the DF contract. It is straightforward to show that the restaurant charges the largest margins under the DD contract, knowing that the platform has two levers to match the drivers' labor supply and online demand. This increases the online price, softening competition between online and offline channels, benefiting only the restaurant. As a response to such an effect, the platform has two contracting choices: to first commit to the margin and relegate online channel pricing to the restaurant (the FD contract), or first commit to the wage offered to the drivers and still control the price in the online channel (the DF contract). Part (i) demonstrates that the DD contract might have an advantage over the DF contract for the platform only when the labor supply cost is pretty high. Note that the DF contract results in the largest supply of drivers, which requires costly investment in labor supply through high wages. Otherwise, both the FD or DF contracts improve the platform's profitability compared to the DD contracts. Furthermore, the platform always prefers FD over DF contracts; the required wage is high in the DF contracts, while the margin charged needs to be low. This boosts online demand at the cost of profit margin as the platform matches pre-committed supply and demand.

Part (ii) characterizes the optimal contracting scheme for the restaurant. The FD contract performs the worst for the restaurant. The reason is that the platform sets a high margin under the FD contract before the restaurant sets the online channel price. Compared to the DD contract, the restaurant has to reduce its margin to increase the online demand and persuade the platform to offer a higher wage for the delivery drivers, given that the platform has only one lever left (i.e., the delivery wage) to match the labor supply of the drivers and the online channel demand. Altogether, the FD contract results in the worst performance for the restaurant. Part (ii) also indicates that the DF or DD contracts might be the best-performing contract for the restaurant. When the cost of supply is not extremely high, the restaurant benefits from the DF contract (see Figure 2), as the high wages committed by the platform ensure an ample supply of drivers, making it profitable for the restaurant to reduce its margin on online orders. As the supply cost significantly increases, providing a large supply becomes particularly challenging for the platform, leading to a substantial decrease in online demand/supply. In this case, the restaurant favors the contract that offers a higher margin, namely the DD contract. However, our extensive numerical analysis in Table 1 shows that the performance of the DD contract only barely surpasses that of the DF contract for the restaurant. This is because, as b decreases to extremely low values, both the DD's advantage over the DF contract in margins and the DF's online demand advantage over the DD would diminish. As a result, while the DD's profit for the restaurant can surpass the DF contract, the benefit is quite negligible.

Figure 2: Comparison of Profits under Different Contracting Schemes



(a) The platform and the restaurant (b) The food delivery chain

From the food delivery chain's perspective, only the DF or the FD contracts can be the best-performing contract. The DF contract often outperforms the other two. Like Part (ii), Figure 2 shows that the FD contract can arise as the best-performing contract for the food delivery chain only when the labor supply cost is high. Our numerical investigation in Table 1 shows that even though the FD contracts dominate the DF contracts for high labor costs, the performance gap between the DF and FD contracts is quite small. Like the DD contract, the FD contract has an online price advantage over the DF contract, while the DF contract has an online demand advantage. As the supply becomes quite costly, these advantages diminish, resulting in somewhat similar performances for the food delivery chain. Unsurprisingly, the DD contract arises as the worst-performing contract for the food delivery chain unless the supply cost is excessively high. As discussed earlier, the DD contract results in the minimum online orders among all contracts, as it cannot coordinate the waging and pricing decisions between the platform and the restaurant. Interestingly, when the restaurant prefers the DD contract over the DF contract, i.e., for extremely costly supply, both the platform and the whole food delivery chain prefer the DD contract over the DF contract, and according to Table 1, the advantage of the DD contract over the DF contract is quite negligible.

Next, we investigate the food delivery chain's online/offline demands and the efficiency of different contracting schemes.

Proposition 5. *We can establish the following:*

(i) For the food delivery chain's demand, we have $q_o^{DF*} + q_f^{DF*} \geq q_o^{FD*} + q_f^{FD*} \geq q_o^{DD*} + q_f^{DD*}$.

(ii) For the demand under the DF contract compared to the centralized case, if $b \leq \frac{1}{2(1-\beta^2)}$*, then* $q_o^{DF*} \geq q_o^{C*}$ *and* $q_o^{DF*} + q_f^{DF*} \geq q_o^{C*} + q_f^{C*}$*; otherwise,* $q_o^{DF*} \leq q_o^{C*}$ *and* $q_o^{DF*} + q_f^{DF*} \leq q_o^{C*} + q_f^{C*}$.

The DF contract allows the restaurant and the platform to coordinate on an intensified competition between the online and offline channels, leading to low online prices and high online orders and increasing the food delivery chain's total demand, as part (i) of Proposition 5 indicates. Interestingly, this contracting scheme can result in an oversupply/excessive demand (compared to the centralized case) in the online channel. In particular, when the supply cost is relatively high, the demand and supply under the DF contract surpass those in a centralized system. Expensive supply indicates that even in a centralized system, investment in supply is low. The DF contract offers a mechanism that allows the platform to increase online demand by committing to a high supply, as we discussed before. The platform finds it optimal to commit more aggressively to supply when the supply cost is high. In contrast, low supply costs indicate that the centralized model invests heavily in the online channel, and the platform does not need to commit to excessive supply under the DF contract.

In a competitive two-sided market, Zhang et al. (2022a) and Hu and Liu (2023) show that wage commitment can intensify market price competition only when the competition intensity on the supply side is greater than that on the demand side. We uncover intensified market competition under the DF contract for a different reason in online food delivery markets, where the sequential nature of decisions by the platform and the restaurant plays an important role, unlike the two-sided markets, where two platforms move simultaneously to set their wages and then prices. The platform moves first under the DF contract in the three-sided food delivery market. The restaurant moves next, and then online and offline channels compete. We show that commitment to a high supply of drivers is a lever for the platform to persuade the restaurant that it would charge competitive delivery fees in the market to keep the online channel competitive, given its committed supply of drivers. Such a commitment coordinates both firms' incentives to charge low margins, making the online channel competitive.

5.2. The Consumers' and the Drivers' Preference over the Contracting Schemes

While we are mainly interested in the food delivery chain's performance, it is also essential to study the effect of these contracting schemes on the other players, i.e., the delivery drivers and the customers. We can find the equilibrium customer and driver surplus (CS^{i*} and DS^{i*}, respectively, for $i \in \{FD, DD, DF\}$), as defined in Equations (B.2) and (B.4) in Online Appendix B.1. Furthermore, social welfare is represented by SW^{i*} (see Equation (B.5) in Online Appendix B.1). The relative performances of the studied contracts are presented in the following proposition.

Proposition 6. *We can establish the following:*

(i) For the customer surplus in equilibrium, we have $CS^{DF*} \geq CS^{FD*} \geq CS^{DD*}$.

(ii) For the driver surplus in equilibrium, we have $DS^{DF*} \geq DS^{FD*} \geq DS^{DD*}$.

(iii) For the social welfare in equilibrium, we have $SW^{DF*} \geq SW^{FD*} \geq SW^{DD*}$.

The customers benefit from the DF contract, as Part (i) of Proposition 6 indicates. Our findings in Proposition 2 help us explain this finding. While the offline price is independent of the contracting schemes, the DF contract results in the lowest online prices, as it coordinates both the restaurant and the platform's incentive to charge lower online prices, which benefits the customers. The customers are worst off under the DD contract as it results in the highest online channel prices and the lowest online demand/supply, which hurts the online customers' surplus.

The findings on the delivery drivers' surplus in Part (ii) of Proposition 6 are unsurprising, as the DF contract not only maximizes the wage but also results in the largest online demand/supply. The commitment to an ample labor supply of drivers in the first stage incentivizes both the restaurant and the platform to price the online channel competitively. This, in turn, boosts online demand, necessitates higher wages, and ultimately increases the drivers' surplus.

Proposition 4 shows that the DF contract usually arises as the dominant contracting scheme for the food delivery chain. Parts (i) and (ii) of Proposition 6 indicate that for both the drivers and customers, this contracting scheme dominates the others; therefore, it is not surprising that this contract maximizes the social welfare among these contracts. The advantage of this contract lies in its capability to coordinate the restaurant and the platform's online channel pricing, pushing for a larger online supply/demand. The only party that loses under this contract is the platform, which prefers the FD contract. Notably, the FD contract performs better than the DD contract for both customers and drivers. The worst performance of the DD contract in providing the right incentive for the restaurant and the platform to coordinate their online pricing hurts both of them, as well as customers and drivers, through high online prices and low delivery wages.

To protect gig economy workers, regulators have extensively discussed establishing minimum wages. For example, NYC has passed a rule requiring online food delivery companies to pay a minimum of \$17.96 per hour before tips to delivery drivers (PYMNTS 2023). Our findings have implications for the regulator. Commitment to a fixed wage (i.e., a minimum wage) that is not excessively large can benefit not only the food delivery chain (in particular, the restaurant) but also customers and drivers. Only the platform loses compared to its best contracting scheme, i.e., the FD contract. To make everyone benefit under the DF contract (compared to the optimal choice for the platform, i.e., the FD contract), a DF contract that includes a subscription fee paid by the restaurant to the platform can help all parties involved in the online food delivery market benefit even under relatively high minimum wages (i.e., for the minimum wage $< w^{DF*}$). Such a contract is guaranteed since the total food delivery chain's profit is maximized under the DF contract unless the supply is pretty expensive, and even if it is not the best-performing contract, it performs quite close to the optimal one. A well-designed transfer from the restaurant to the platform can make the platform indifferent between the FD and the newly designed contract. In contrast, the restaurant prefers the new contract to the FD contract, which minimizes its profit. Such a transfer payment does not affect the equilibrium outcomes and, therefore, customer and driver surplus. In practice, some platforms like Sesame have started to charge restaurants fixed monthly fees (Joe 2021). Moreover, this contract interests platforms trying to increase their market share, as it allows the food delivery chain to increase online sales while benefiting both the platform and the restaurant.

The above discussion shows that a relatively high minimum wage might be beneficial not only for the drivers but also for society. In Section 7, we show that a relatively high minimum wage rate has different implications than the minimum wage commitment for the food delivery chain. This reveals an interesting observation for regulators designing new minimum wage or wage rate regulations. Next, the following section uses extensive numerical experiments to improve our understanding of the contracting schemes studied and their relative performances.

6. Numerical Experiments

In this section, we use extensive numerical experiments to shed light on the relative performance of the studied contracting schemes for the platform, the restaurant, and the food delivery chain. To achieve this goal, we have considered the following ranges for the model parameters: $\beta \in [0.01, 1]$, $a \in [0.01, 1]$, and $b \in [0.1, 1.9]$. We divide the ranges for β and a (b) into 100 (10) equally placed intervals and use a combination of these values to solve for the optimal contracting terms for all the contracting schemes and the centralized case. We also assess the feasibility of these contracting terms, ensuring that both the online and offline channels remain active in equilibrium. In total, we analyze 44,458 different feasible scenarios.

We denote the loss of profit for firm f in scheme t compared to scheme k as $L_f^{t,k} = \frac{\pi_f^k - \pi_f^t}{\pi_f^k}$, with $\pi_f^t < \pi_f^t$. Here $f \in \{r, p, sc\}$ and $t, k \in \{FD, DD, DF, C\}$. Tables 1 summarizes our findings. It demonstrates that the DF contract performs well for the food delivery chain compared to the centralized solution, as the loss of profit stands at 0.54% on average, with a maximum of 4.12% among all tested scenarios. The performance of the next best contract, i.e., the FD contract, is also good at 1.05% loss of profit on average. The loss of profit for the DD contract is more significant, averaging at 2.09%.

We use this table to illustrate that while the DF contract may not always be the optimal choice for the food delivery chain or for the restaurant, as shown in Proposition 4, the profit loss is relatively minor compared to the best-performing contracts. If the supply chain profit under the DF contract is less than that under the FD contract, the average profit loss is 0.1%. Additionally, the profit loss for both the supply chain and the restaurant under the DF contract compared to the DD contract is quite negligible. Therefore, we claim that the DF contract arises as the preferred contract for both the food delivery chain and the restaurant.

Table 1: The Performance of Different Contracting Schemes

Loss of profit	Average	10 percentile	90 percentile	Maximum
$L_{sc}^{DF,C}$	0.0054	0.0000	0.0165	0.0412
$L_{sc}^{FD,C}$	0.0105	0.0001	0.0287	0.0609
$L_{sc}^{DD,C}$	0.0209	0.0005	0.0533	0.0971
$L_{sc}^{DF,FD}$	0.0011	0.0000	0.0026	0.0215
$L_{sc}^{FD,DF}$	0.0055	0.0002	0.0135	0.0234
$L_{sc}^{DF,DD}$	0.0008	0.0000	0.0028	0.0052
$L_{r}^{DF,DD}$	0.0006	0.0000	0.0019	0.0035
$L_{r}^{DD,DF}$	0.0112	0.0003	0.0279	0.0514

Table 2: Demand Gap for Different Contracting Schemes

Demand Gap	Average	Minimum	10 percentile	90 percentile	Maximum
$K_{p}^{DF,C}$	0.2124	0	0.0661	0.3077	0.8899
$K_{p}^{FD,C}$	0.3205	0.0190	0.2294	0.3853	0.3958
$K_{p}^{DD,C}$	0.5	0.5	0.5	0.5	0.5
$K_{r}^{DF,C}$	0.0284	0	0.0011	0.0714	0.1269
$K_{r}^{FD,C}$	0.0416	0	0.0032	0.0939	0.1543
$K_{r}^{DD,C}$	0.0596	0	0.0067	0.1268	0.1949

In addition to the profit loss, we also study the demand gap between each contract and the centralized case, focusing on both the online demand and total demand. We define the demand gap in contract scheme t compared to the centralized case C as $K_g^{t,C} = \frac{|Q_g^C - Q_g^t|}{Q_g^C}$, where $g \in \{o, sc\}$ and $t \in \{FD, DD, DF\}$, and Q_g denotes the demand for the online channel o or the total supply chain sc. Table 2 illustrates the resutls. Despite its prevalence in practice, it is noteworthy that the DD contract induces 50% less capacity than the centralized capacity. This also justifies the poor performance of this contract compared to the centralized case. The online demand gap with the centralized case reduces significantly as the platform adopts the FD or DF contracts. It is also notable that even the DF contract results in about 21.24% loss/excess in online demand. For the total demand, the change in offline demand moderates the changes in total demand under the DF contract, as the total demand loss/excess stands at only 2.84%. Such a small gap in total demand justifies our earlier finding that the DF contract can achieve more than 99.5% of the total profit for the food delivery chain (on average). The lowest demand loss/excess under the DF contract indicates better coordination between the restaurant and the platform to serve online customers compared to the centralized case.

7. Extensions

In this section, we extend our model in two directions to examine the robustness of our findings. One direction accounts for the utility of drivers who provide delivery service, and the other investigates the robustness of our findings concerning the demand models for the online and offline channels.

7.1. Utility Framework for Drivers

The labor supply model of delivery drivers used in the main body of this study has a limitation in that it only considers the supply as a function of the wage offered by the platform. In other words, it assumes that the platform pays the delivery drivers a certain wage per unit of time. In practice, it is also common for drivers to get paid based on the number of deliveries they make. Therefore, these drivers' motivation to participate in food delivery is also a function of the online channel demand rate (i.e., how busy they are with deliveries). While the supply model of the main model captures the main characteristics of the labor supply of the drivers, it does not explicitly account for the impact of the online demand rate on the supply of the delivery drivers. In the rest of this subsection, we present a framework to address this issue and demonstrate the robustness of our findings in the main model. Moreover, we reveal the implications of introducing the wage rate into the labor supply model for the regulators designing mechanisms to protect the gig economy workers (i.e., the drivers in our model).

Gig workers are in high demand, with platforms competing to hire them. They can choose when and where to work based on the wages offered (Zhang et al. 2022a). As a result, many gig drivers work for multiple platforms simultaneously and can switch between them in real-time. To model this phenomenon, we follow the literature on differentiated duopolies (Abhishek et al. 2016, Sun et al. 2023) and use a utility framework similar to Equation (B.3) (in Online Appendix B) to model the drivers' choice. Specifically, a representative driver can work for the platform or an outside option. If he chooses to work for the outside option, he will receive a fixed amount of income per unit of time y, while if he works for the platform, his expected wage is given by wq_o, i.e., the wage rate paid by the platform times the online channel demand. Assume that the representative driver maximizes the following utility function,

$$\arg\max_{l_p,l_o} U(wq_o, y) = l_p w q_o + l_o y - \tfrac{1}{2}l_p^2 - \tfrac{1}{2}l_o^2 - \phi l_p l_o, \tag{33}$$

where l_p and l_o denote the amount of labor that a representative driver allocates to work for the platform and the outside option, respectively, and ϕ represents the substitutability of working for the platform vs. the outside option. Assuming that the thickness of the drivers' supply market is N, it is straightforward to show that the supply of drivers for the platform is given by

$$s = \tfrac{wq_o - \phi y}{1-\phi^2} N. \tag{34}$$

The above formulation of drivers' labor supply captures the indirect network effect of online demand on the supply side. In particular, increasing online demand also implies a greater interest from drivers to work for the platform. This feature is mainly overlooked in the main supply model, given by Equation (3). Substituting the supply equation, we solve for the equilibrium market outcomes under the three contracting schemes represented in the following proposition. All the proofs are available from the authors.

PROPOSITION 7. *If the labor supply of the delivery drivers is given by* (34)*, we have,*

(i) The online channel price satisfies $p_o^{DD*} \geq p_o^{FD*} \geq p_o^{DF*}$.

(ii) The wage rate for the delivery drivers satisfies $w^{DF*} \leq w^{FD*} \leq w^{DD*}$*, but the wage paid to delivery drivers satisfies* $q_o^{DF*} w^{DF*} \geq q_o^{FD*} w^{FD*} \geq q_o^{DD*} w^{DD*}$.

(iii) The online channel demand satisfies $q_o^{DF*} \geq q_o^{FD*} \geq q_o^{DD*}$.

Proposition 7 confirms that introducing wage rates does not impact our main findings. In particular, while the wage rate under the DF contract is the minimum among all schemes, the total wage paid to delivery drivers that incorporate demand rates (i.e., wq_o) is still the maximum under the DF contract. This finding also aligns with Parts (i) and (iii), confirming that the wage rate-induced labor supply does not change the conclusions of the main model. In particular, the DD contract has the highest online channel price, while the DF contract results in the most fierce market competition. The intuition behind these findings is quite the same as that of the main model.

The mechanism in which the DF contract delivers its good performance somehow differs between the main and the wage rate models. When supply is only a function of the wage (i.e., the main model), the platform has to commit to a high wage in the first stage to induce enough participation by the drivers. However, when driver supply is a function of the wage and the demand rate together, the platform should commit to a low enough wage rate, inducing the restaurant to reduce the margin to provide the right incentive for the platform to post low delivery fees (knowing that the platform pays a low wage rate); this aligns the platform and the restaurant's incentive to make the online channel competitive against the dine-in offline channel, by setting low online prices.

Next, we investigate how the introduction of the wage rate affects the players' profitability in the food delivery chain under different contracting schemes.

PROPOSITION 8. *If the supply of the delivery drivers is given by* (34)*, we have,*

(i) If $0 < N \leq \frac{1-\phi^2}{8-8\beta^2}$*, then* $\pi_p^{FD*} \geq \pi_p^{DD*} \geq \pi_p^{DF*}$*; otherwise,* $\pi_p^{FD*} \geq \pi_p^{DF*} \geq \pi_p^{DD*}$.

(ii) If $0 < N \leq \frac{1-\phi^2}{8-8\beta^2}$*, then* $\pi_r^{DD*} \geq \pi_r^{DF*} \geq \pi_r^{FD*}$*; otherwise,* $\pi_r^{DF*} \geq \pi_r^{DD*} \geq \pi_r^{FD*}$.

(iii) If $0 < N \leq \frac{1-\phi^2}{8-8\beta^2}$*, then* $\pi_{sc}^{FD*} \geq \pi_{sc}^{DD*} \geq \pi_{sc}^{DF*}$*; if* $\frac{1-\phi^2}{8-8\beta^2} < N \leq \frac{(\sqrt{33}-1)(\phi^2-1)}{16(\beta^2-1)}$*, then* $\pi_{sc}^{FD*} \geq \pi_{sc}^{DF*} \geq \pi_{sc}^{DD*}$*; otherwise,* $\pi_{sc}^{DF*} \geq \pi_{sc}^{FD*} \geq \pi_{sc}^{DD*}$.

(iv)For the customer's surplus in equilibrium: $CS^{DF*} \geq CS^{FD*} \geq CS^{DD*}$.

(v)For the driver's surplus in equilibrium: $DS^{DF*} \geq DS^{FD*} \geq DS^{DD*}$.

While the utility formulation of the drivers' supply is quite different from the supply formulation in (3), Proposition 8 shows that our findings in the main model are pretty robust. In particular, Proposition 8 indicates that when the supply market is pretty thin (i.e., $N \leq \frac{1-\phi^2}{8-8\beta^2}$), which indicates raising supply is excessively costly, the platform prefers the FD contract while the restaurant prefers the DD contract. A thicker market, as it implies a cheaper labor supply, makes the restaurant prefer the DF contract. Proposition 8 also characterizes how different contracting schemes affect the drivers and customers. In particular, we can show that the DF contract with fixed wage rates might benefit not only the restaurant but also the drivers and customers, which aligns with our previous findings. Such a contract resembles the regulator's move to set a minimum wage for the delivery drivers per unit of time spent for actual deliveries. For example, the State Court of New York has ruled that the food delivery platforms should pay the delivery drivers 50 cents per minute of delivery before tips (Lindeque 2024). It is important to note that as Part (ii) of Proposition 7 implies, under DF contracts with fixed wage rates, while the platform commits to a low wage rate, this commitment to the wage rate allows both the restaurant and the platform to coordinate on charging low margins to keep the online channel competitive vs. the offline channel in the food market. Competitive online pricing increases online orders, maximizing the wage drivers receive under the DF with fixed wage rates compared to the other contracts. Notably, suppose the regulator sets a minimum wage rate $\underline{w}$ that exceeds w^{DF*} ($\underline{w} \geq w^{DF}$). In that case, such regulations can hurt the drivers, the food delivery chain, and online customers through higher online channel prices and lower online orders. To conclude, while a relatively high minimum wage can benefit the drivers and the food delivery chain, a relatively high wage rate can hurt the drivers and the whole food delivery chain.

7.2. An Alternative Demand Model

As mentioned earlier, one of the features of the online and offline channel demands in our main model is that the total market size increases as the two channels become more differentiated. However, this feature might only sometimes hold. In this subsection, we study a demand model that assumes the total potential market size is independent of the degree of differentiation between the channels. In particular, we adopt the model of Raju et al. (1995) and assume the online and offline channel demands as

$$q_o = 1 - p_o + \beta(p_f - p_o), \tag{35}$$
$$q_f = \alpha - p_f + \beta(p_o - p_f), \tag{36}$$

where α represents the relative potential size of the dine-in offline vs. the online channel. It is essential to investigate the robustness of our findings as α changes. In particular, when $\alpha \leq 1$, the online channel potential can be larger than that of the dine-in offline channel. In contrast, $\alpha > 1$ implies a larger potential market size for the dine-in channel that customers prefer to get served at the restaurant. Substituting the demand model in (1) and (2) by (35) and (36), respectively, we can establish Proposition B1 in Online Appendix B.2.

Proposition B1 shows that our findings in the main model still hold as we incorporate different market potentials for the online and offline channels. In particular, the most fierce channel competition happens under the DF contract, while the DD contract softens channel competition. Similar to our findings in Proposition 8, as long as raising the supply of drivers is not excessively expensive (i.e., when b is not very low), the platform prefers the FD contract. In contrast, the restaurant prefers the DF contract. Moreover, if working for the platform is attractive enough (i.e., b is large enough), the food delivery chain would benefit from the DF contract through larger online orders. Altogether, our findings in this extension subsection demonstrate that the relative market size parameter α does not play a significant role in the relative performance of these contracts.

8. Discussion

The emergence of the sharing economy has prompted the evolution of digital platforms, which have yet to be extensively studied in the literature. This paper addresses this gap by examining online food delivery platforms' pricing and waging decisions within a three-sided market. We provide an analytical framework focusing on the key trade-offs a food delivery platform encounters as it contracts with restaurants and gig economy drivers to provide food delivery services to customers.

In such a three-sided online food delivery market, the match of online demand and labor supply of drivers requires careful management of the platform's relationship not only with the self-scheduling drivers but also with restaurants, as they are providing the food offered on the platform. Without self-scheduling drivers, we show that all the introduced contracting schemes have the same market outcome for the food delivery chain. This observation falls apart as we incorporate the self-scheduling nature of the delivery drivers' supply for online platforms. All contracts result in different market outcomes, highlighting the importance of modeling a three-sided market. Compared to the traditional economy, resorting to self-scheduling service providers could soften market competition if the traditional economy suffers from insufficient capacity.

Under the sharing economy, the platform always prefers the FD contract, while the restaurant and the whole food delivery chain usually prefer the DF contract. In the FD contract, the platform asks for large margins, knowing that the restaurant has to moderate its margin because it is the platform that controls the labor supply through the wages offered to drivers. The DF contract maximizes the online demand by coordinating the platform and the restaurant's incentives to charge low margins. While the platform and the restaurant move sequentially, commitment to a high wage aligns both the platform's and the restaurant's incentives to offer competitive online prices. Given the inferior performance of the DF contract for the platform, we propose a subscription fee to compensate the platform if it decides to implement the DF contract, so it would benefit not only the food delivery chain but also customers and drivers.

Finally, we investigate different implementations of the minimum wage requirement contemplated by regulators to protect drivers and increase social welfare. Given the optimality of the DF contract for the food delivery chain, maximizing the driver's and customers' surplus and social welfare, we show that a relatively high minimum wage can benefit all of them. In contrast, if the regulator aims to control the minimum wage rate, then a relatively low rate can protect the drivers while benefiting social welfare and the food delivery chain's profit, as it coordinates the platform and the restaurant's incentives to increase the competitiveness of the online channel.

This paper only models the competition between a single restaurant's online food delivery and offline dine-in channels. Therefore, future research should investigate the robustness of the findings when there is competition among multiple restaurants and platforms. Competition among these firms can introduce new driving forces to shape the equilibrium outcomes. Another line of future research can look for contracts that improve the entire food delivery chain's performance.

References

Abhishek V, Jerath K, Zhang ZJ (2016) Agency selling or reselling? channel structures in electronic retailing. *Management Science* 62(8):2259–2280.

Andrei H (2009) Two-sided platforms: Product variety and pricing structures. *Journal of Economics & Management Strategy* 18(4):1011–1043.

Bahrami S, Nourinejad M, Yin Y, Wang H (2023) The three-sided market of on-demand delivery. *Transportation Research Part E: Logistics and Transportation Review* 179:103313.

Banerjee S, Riquelme C, Johari R (2016) Pricing in ride-share platforms: A queueing-theoretic approach. *Working paper* URL `https://ssrn.com/abstract=2568258`.

Cachon GP, Daniels KM, Lobel R (2017) The role of surge pricing on a service platform with self-scheduling capacity. *Manufacturing & Service Operations Management* 19(3):368–384.

Caillaud B, Jullien B (2003) Chicken & egg: Competition among intermediation service providers. *The RAND Journal of Economics* 34(2):309–328.

Chen M, Hu M (2024) Courier dispatch in on-demand delivery. *Management Science* 70(6):3789–3807.

Chen M, Hu M, Wang J (2022) Food delivery service and restaurant: Friend or foe? *Management Science* 68(9):6539–6551.

Christopher P (2023) Doordash partnership plans give merchants more options. URL `https://get.doordash.com/en-us/blog/new-partnership-plans`.

Doordash (2023a) Commission and fees on doordash, explained. URL `https://get.doordash.com/en-au/learning-center/delivery-commission`.

Doordash (2023b) How to add storefront to your website? URL `https://get.doordash.com/en-us/learning-center/grow-online-with-storefront`.

Eleme (2024) Explanation of order amount calculation on the ele.me food service provider platform. URL `https://open.shop.ele.me/base/documents/orderamount`.

Feldman P, Frazelle AE, Swinney R (2023) Managing relationships between restaurants and food delivery platforms: Conflict, contracts, and coordination. *Management Science* 69(2):812–823.

Garg N, Nazerzadeh H (2022) Driver surge pricing. *Management Science* 68(5):3219–3235.

Hu M, Liu Y (2023) Precommitments in two-sided market competition. *Manufacturing & Service Operations Management* 25(2):704–718.

Hu M, Zhou Y (2020) Price, wage, and fixed commission in on-demand matching. *Working paper* URL https://ssrn.com/abstract=2949513.

IMARC Group (2024) Online food delivery market report by platform type, business model, payment method, and region 2024-2032. URL https://www.imarcgroup.com/online-food-delivery-market.

Jerath K, Zhang ZJ (2010) Store within a store. *Journal of Marketing Research* 47(4):748–763.

Joe G (2021) Commission-free delivery startup sesame raises \$3.4m. URL https://www.restaurantbusinessonline.com/technology/commission-free-delivery-startup-sesame-raises-34m.

Karamshetty V, Freeman M, Hasija S (2023) Select, swipe, and serve: Examining the impact of food-delivery platforms on restaurant demand characteristics. *Working paper* URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3667539.

Kreps DM, Scheinkman JA (1983) Quantity precommitment and bertrand competition yield cournot outcomes. *The Bell Journal of Economics* 14(2):326–337.

Li Z, Wang G (2024a) On-demand delivery platforms and restaurant sales. *Management Science, Forthcoming* URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3813891.

Li Z, Wang G (2024b) Regulating powerful platforms: Evidence from commission fee caps. *Information Systems Research, Forthcoming* URL http://dx.doi.org/10.1287/isre.2022.0191.

Lindeque M (2024) Food delivery drivers to be paid by the minute. URL https://www.locate2u.com/delivery-drivers/food-delivery-drivers-to-be-paid-by-the-minute/.

Liu Z, Shen Y, Sun Y (2023) Operating three-sided marketplace: Pricing and spatial staffing in food delivery systems. *Working paper* URL https://ssrn.com/abstract=4668867.

Mao W, Ming L, Rong Y, Tang CS, Zheng H (2019) Faster deliveries and smarter order assignments for an on-demand meal delivery platform. *Working paper* URL https://ssrn.com/abstract=3469015.

Oh J, Glaeser C, Su X (2023) Food ordering and delivery: How platforms and restaurants should split the pie. *Working paper* URL https://ssrn.com/abstract=4366228.

PYMNTS (2023) New york court rules food delivery platforms must pay minimum wage. URL https://www.pymnts.com/news/delivery/2023/new-york-court-rules-food-delivery-platforms-must-pay-minimum-wage/.

Raju JS, Sethuraman R, Dhar SK (1995) The introduction and performance of store brands. *Management Science* 41(6):957–978.

Rochet JC, Tirole J (2003) Platform competition in two-sided markets. *Journal of the European Economic Association* 1(4):990–1029.

Singh N, Vives X (1984) Price and quantity competition in a differentiated duopoly. *The RAND Journal of Economics* 15(4):546–554.

Sun G, Kim Y, Tan Y, Parker GG (2023) Dinner at your doorstep: Service innovation via the gig economy on food delivery platforms. *Information Systems Research, Ahead of Print* .

Taylor TA (2018) On-demand service platforms. *Manufacturing & Service Operations Management* 20(4):704–720.

Tirole J (1988) *The theory of industrial organization* (MIT press).

Zhang C, Chen J, Raghunathan S (2022a) Two-sided platform competition in a sharing economy. *Management Science* 68(12):8909–8932.

Zhang W, Tang CS, Ming L, Cheng Y (2022b) Reducing traffic incidents in meal delivery: Penalize the platform or its independent drivers? *Working paper* URL `https://ssrn.com/abstract=4231746`.

Zhang W, Zheng Z, Cui R (2023) Restaurant density and delivery speed in food delivery platforms. *Working paper* URL `https://ssrn.com/abstract=4419279`.

Online Appendix to "Online Food Delivery Contracting in Three-Sided Markets"

Appendix A: Notations

Table A.1: Table of Notations

Symbols	Description
β	the degree of differentiation between online and offline channels
a	attraction of outside work options for drivers
b	delivery drivers' sensitivity to the platform's wage
p_f	offline (full) market price
p_o	online (full) market price
m_p	online profit margin of the platform
m	online profit margin of the restaurant
d	delivery fee
r	commission fee asked by the platform
w	driver's wage
q_o	online demand
q_f	offline (dine-in) demand
s	supply of delivery drivers
π_f	firm f's profit, $f \in \{p, r, sc\}$
$L_f^{t,k}$	loss of profit for firm f in scheme t compared to scheme k, where $f \in \{p, r, sc\}$, and $t, k \in \{FD, DD, DF, C\}$
$K_g^{t,C}$	demand gap in scheme t compared to the centralized case, where $g \in \{o, sc\}$, and $t \in \{FD, DD, DF\}$
l_o	the amount of labor that the driver allocates to work for the outside option
l_p	the amount of labor that the driver allocates to work for the platform
ϕ	the substitutability of working for the platform vs. the outside option
y	fixed amount of income per unit of time for outside option
N	The thickness of drivers' supply
α	relative potential market size of the dine-in channel vs. online channel

Appendix B: Additional Analytical Results

B.1. Drivers/Customer's Surplus

To derive the delivery drivers' surplus while working for the platform, we apply the classical theory of monopoly markets (e.g., Tirole 1988). The surplus is given by

$$DS^i = \int_0^{-a+bw^i} (w^i - \tfrac{s+a}{b})\, ds = \tfrac{(-a+bw^i)^2}{2b}, \tag{B.1}$$

as a function of the supply parameters (a, b) and the equilibrium wages under the contracting scheme $i \in \{FD, DD, DF\}$. Substituting optimal wage w^{i*} into Equation (B.1), we can get

$$\begin{aligned} DS^{FD*} &= \tfrac{(a+b(\beta-1))^2}{8b(1-2b(\beta^2-1))^2}, \\ DS^{DD*} &= \tfrac{(b(1-\beta)-a)^2}{32b(-b\beta^2+b+1)^2}, \\ DS^{DF*} &= \tfrac{(a+b(\beta-1))^2}{2b(1-4b(\beta^2-1))^2}. \end{aligned} \tag{B.2}$$

The market demand functions (1) and (2) follow from the consumption quadratic utility of a representative customer:

$$U(q_o, q_f, p_o, p_f) = q_o + q_f - \tfrac{1}{2}q_o^2 - \tfrac{1}{2}q_f^2 - \beta q_o q_f - p_o q_o - p_f q_f. \tag{B.3}$$

Hence, the customer's surplus under each contract is equivalent to the customer's utility, that is, $CS^i = U^i(q_o^i, q_f^i, p_o^i, p_f^i)$, $i \in \{FD, DD, DF\}$. Substituting optimal channel prices and demands p_o^{i*}, p_f^{i*}, q_o^{i*}, q_f^{i*} into Equation (B.3), we can get

$$\begin{aligned} CS^{FD*} &= \tfrac{a^2(1-\beta^2)-2ab(\beta-1)^2(\beta+1)+b(\beta-1)(\beta+1)(b(\beta-1)(3\beta+5)-4)+1}{8(1-2b(\beta^2-1))^2}, \\ CS^{DD*} &= \tfrac{a^2(1-\beta^2)-2ab(\beta-1)^2(\beta+1)+b(\beta-1)(\beta+1)(b(\beta-1)(3\beta+5)-8)+4}{32(-b\beta^2+b+1)^2}, \\ CS^{DF*} &= \tfrac{4a^2(1-\beta^2)-8ab(1-\beta)^2(\beta+1)+4b(1-\beta^2)(b(1-\beta)(3\beta+5)+2)+1}{8(1-4b(\beta^2-1))^2}. \end{aligned} \tag{B.4}$$

In addition, we define the total social welfare as follows ($i \in \{FD, DD, DF\}$):

$$SW^i = \pi_p^i + \pi_r^i + DS^i + CS^i. \tag{B.5}$$

B.2. Results of the Alternative Demand Model

Following the analysis in Section 4, we can derive the equilibrium solutions for each type of contract, which we omit here. Please contact the authors for detailed information. We then investigate how the introduction of the alternative demand model affects the player's profitability in the food delivery chain under different contracting schemes. Results are shown in the following proposition.

PROPOSITION B1. *We can establish the following when the demand function is given by* (35) *and* (36)*.*

(i) The online channel price satisfies $p_o^{DD*} \geq p_o^{FD*} \geq p_o^{DF*}$.

(ii) The delivery drivers wage satisfies $w^{DF*} \geq w^{FD*} \geq w^{DD*}$.

(iii) The online channel demand satisfies $q_o^{DF*} \geq q_o^{FD*} \geq q_o^{DD*}$.

(iv) For the equilibrium profit of the platform, if $b \geq \frac{1+\beta}{8}$*, then* $\pi_p^{FD*} \geq \pi_p^{DF*} \geq \pi_p^{DD*}$*; otherwise,* $\pi_p^{FD*} \geq \pi_p^{DD*} \geq \pi_p^{DF*}$.

(v) For the equilibrium profit of the restaurant, if $b \geq \frac{1+\beta}{8}$*, then* $\pi_r^{DF*} \geq \pi_r^{DD*} \geq \pi_r^{FD*}$*; otherwise,* $\pi_r^{DD*} \geq \pi_r^{DF*} \geq \pi_r^{FD*}$.

(vi) For the equilibrium profit of the supply chain, if $b \geq \frac{(\sqrt{33}-1)(\beta+1)}{16}$, $\pi_{sc}^{DF*} \geq \pi_{sc}^{FD*} \geq \pi_{sc}^{DD*}$*; if* $\frac{1+\beta}{8} \leq b < \frac{(\sqrt{33}-1)(\beta+1)}{16}$*, then* $\pi_{sc}^{FD*} \geq \pi_{sc}^{DF*} \geq \pi_{sc}^{DD*}$*; otherwise,* $\pi_{sc}^{FD*} \geq \pi_{sc}^{DD*} \geq \pi_{sc}^{DF*}$.

Appendix C: Proofs of Main Results

C.1. Proof of Lemma 1

The total amount of supply is defined as $s = -a + bw$ in Equation (3). Equivalently, this can be expressed as $w = \frac{a+s}{b}$. To simplify our analysis, we consider the scenario where the centralized decision maker determines the supply s. We then examine two cases: $s \geq q_o$ and $s \leq q_o$, respectively.

(i) In the region where $s \geq q_o$, a centralized decision maker maximizes its following profit:

$$\begin{aligned} \Pi^C = &\max_{s, p_o, p_f} q_o\left(p_o - \tfrac{a+s}{b}\right) + p_f q_f, \\ &s.t. \quad s \geq q_o. \end{aligned}$$

First, we can show that the centralized decision maker's profit decreases with s given any p_o and p_f as $\frac{\partial \Pi^C}{\partial s} = -\frac{q_o}{b} < 0$. Thus, the centralized decision maker decreases s until $s = q_o$. Substituting $s = q_o$ into its profit, we can easily find that its profit is jointly concave in p_o and p_f because the Hessian matrix is negative definite. Specifically, the Hessian matrix of Π^C with respect to p_o and p_f are given by

$$H=\begin{bmatrix}\frac{\partial^2\Pi^C}{\partial p_o^2} & \frac{\partial^2\Pi^C}{\partial p_o\partial p_f}\\ \frac{\partial^2\Pi^C}{\partial p_f\partial p_o} & \frac{\partial^2\Pi^C}{\partial p_f^2}\end{bmatrix}=\begin{bmatrix}\frac{2b(\beta^2-1)-2}{b(\beta^2-1)^2} & \frac{2\beta(-b\beta^2+b+1)}{b(\beta^2-1)^2}\\ \frac{2\beta(-b\beta^2+b+1)}{b(\beta^2-1)^2} & \frac{2(b-1)\beta^2-2b}{b(\beta^2-1)^2}\end{bmatrix}.$$

Hence, the optimal online channel price p_o and the offline channel price p_f satisfy $\frac{\partial\Pi^C}{\partial p_o}=0$, $\frac{\partial\Pi^C}{\partial p_f}=0$, simultaneously, and are characterized by $p_o^*=\frac{(1-\beta^2)(a+b)+2-\beta}{2b(1-\beta^2)+2}$ and $p_f^*=\frac{1}{2}$. Substituting p_o^* and p_f^* back to $s=q_o$, we obtain $s^*=\frac{a+b(\beta-1)}{2b(\beta^2-1)-2}$, and correspondingly, $w^*=\frac{a+b(1-\beta)+2ab(1-\beta^2)}{2b(1+b(1-\beta^2))}$.

(ii) In the region where $s\le q_o$, a centralized decision maker maximizes its following profit:

$$\Pi^C=\max_{s,p_o,p_f} s(p_o-\tfrac{a+s}{b})+p_f q_f,\qquad s.t.\quad s\le q_o.$$

First, we can show that the centralized decision maker's profit increases with p_o given any s and p_f as $\frac{\partial\Pi^C}{\partial p_o}=\frac{(\beta^2-1)s-\beta p_f}{\beta^2-1}>0$. Additionally, we find that as p_o increases, online demand decreases because $\frac{\partial q_o}{\partial p_o}=\frac{1}{\beta^2-1}<0$. Hence, the centralized decision maker increases p_o until $q_o=s$, at which $p_o=\beta(p_f-1)+(\beta^2-1)s+1$. Substituting $p_o=\beta(p_f-1)+(\beta^2-1)s+1$ into its profit, we can easily show that its profit is jointly concave in s and p_f as the Hessian matrix is negative definite. Specifically, the Hessian matrix of Π^C with respect to s and p_f are given by

$$H=\begin{bmatrix}\frac{\partial^2\Pi^C}{\partial s^2} & \frac{\partial^2\Pi^C}{\partial s\partial p_f}\\ \frac{\partial^2\Pi^C}{\partial p_f\partial s} & \frac{\partial^2\Pi^C}{\partial p_f^2}\end{bmatrix}=\begin{bmatrix}2\beta^2-\frac{2}{b}-2 & 0\\ 0 & -2\end{bmatrix}.$$

Hence, the optimal supply s and the offline channel price p_f satisfy $\frac{\partial\Pi^C}{\partial s}=0$ and $\frac{\partial\Pi^C}{\partial p_f}=0$ simultaneously, and are characterized by $s^*=\frac{a+b(\beta-1)}{2b(\beta^2-1)-2}$ and $p_f^*=\frac{1}{2}$. Substituting s^* and p_f^* back to p_o, we can get $p_o^*=\frac{(1-\beta^2)(a+b)+2-\beta}{2b(1-\beta^2)+2}$, and correspondingly, $w^*=\frac{a+b(1-\beta)+2ab(1-\beta^2)}{2b(1+b(1-\beta^2))}$.

In summary, we find Case (i) and Case (ii) ($s\ge q_o$ and $s\le q_o$) lead to the same equilibrium results shown in Lemma 1. Consequently, we have the following equilibrium quantities and profits:

$$\begin{aligned}&q_o^{C*}=s^{C*}=\tfrac{b(1-\beta)-a}{2b(1-\beta^2)+2};\quad q_f^{C*}=\tfrac{a\beta+b(1-\beta)+1}{2b(1-\beta^2)+2};\\ &\Pi^{C*}=\tfrac{a^2-2ab(1-\beta)+2b^2(1-\beta)+b}{4b(b(1-\beta^2)+1)}.\end{aligned}\tag{C.1}$$

□

C.2. Proof of Lemma 2

In this proof section, we first detail the firm's profit under each sub-case and then solve for the equilibrium decisions in the BFD, BDD, and BDF contracts, respectively. Subsequently, we compare the equilibrium outcomes of these three contracts.

Fixed-price/dynamic-wage contract (BFD). Under this contract, the platform first announces its commission fee of r to the restaurant. Then, the restaurant sets the dine-in channel price p_f and the online channel price p_o by setting the online profit margin m. In this sub-case, the profits of both the platform and the restaurant are given by:

$$\pi_p^{BFD}(r)=\min(\hat{s},q_o)(r-c),\tag{C.2}$$
$$\pi_r^{BFD}(m,p_f)=\min(\hat{s},q_o)m+p_f q_f,\tag{C.3}$$

where $\min(\hat{s}, q_o)$ denotes the online sales, with $\hat{s}$ being exogenously given. In this case, the platform's margin is given by $m_p^{BFD} = r$. Additionally, the online channel price is the sum of the commission fee charged by the platform and the margin set by the restaurant and is given by $p_o = m + r$.

Dynamic-price/dynamic-wage (BDD). Under this contract, the platform first announces its commission fee r to the restaurant. Then, the restaurant sets its dine-in channel price alongside the online sales margin, m for each online order. Finally, the platform determines the online channel price by setting the delivery fee d for online orders. The platform and restaurant's profit are given by

$$\pi_p^{BDD}(r, d) = \min(\hat{s}, q_o)(r + d - c), \tag{C.4}$$
$$\pi_r^{BDD}(m, p_f) = \min(\hat{s}, q_o)m + p_f q_f. \tag{C.5}$$

Here, the platform's margin $m_p^{BDD} = r + d$ and the online channel price is $p_o = m + r + d$.

Dynamic-price/fixed-wage contract (BDF). Since the wage is exogenously fixed at a constant c, the BDF contract follows the same decision sequence as the BDD contract. Specifically, the platform first announces the commission fee, followed by the restaurant setting its margin for online orders and dine-in channel prices. Finally, the platform determines the delivery fee. As a result, the profits for the platform and the restaurant are given by Equations (C.4) and (C.5).

We then solve for the equilibrium decisions in the BFD, BDD, and BDF contracts, respectively.

Fixed-price/dynamic-wage contract (BFD). Using backward induction, we first solve for the restaurant's optimization problem:

$$\pi_r^{BFD} = \max_{m, p_f} \min(\hat{s}, q_o)m + p_f q_f,$$

where the restaurant's online margin is $m = p_o - r$ with r fixed in this stage. Hence, we can equivalently solve the problem where the restaurant's decision is online channel price p_o. In this benchmark, the exogenous supply ($\hat{s}$) may either exceed or fall short of demand, leading us to consider the following two cases.

(i) When supply exceeds demand ($\hat{s} \geq q_o$), the restaurant's optimization problem becomes

$$\begin{aligned} \pi_r^{BFD} = & \max_{p_o, p_f} q_o(p_o - r) + p_f q_f, \\ s.t. \quad & \hat{s} \geq q_o. \end{aligned} \tag{C.6}$$

The restaurant's profit is jointly concave in p_o and p_f because the Hessian matrix is negative definite. The Hessian matrix is given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{BFD}}{\partial p_o^2} & \frac{\partial^2 \pi_r^{BFD}}{\partial p_o \partial p_f} \\ \frac{\partial^2 \pi_r^{BFD}}{\partial p_f \partial p_o} & \frac{\partial^2 \pi_r^{BFD}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{2}{\beta^2 - 1} & \frac{2\beta}{1 - \beta^2} \\ \frac{2\beta}{1 - \beta^2} & \frac{2}{\beta^2 - 1} \end{bmatrix}.$$

By applying KKT(Karush-Kuhn-Tucker) conditions, we can rewrite problem (C.6) as follows.

$$\begin{aligned} \mathcal{L}(p_o, p_f, \lambda) = & \; q_o(p_o - r) + p_f q_f + \lambda(\hat{s} - q_o), \\ s.t. \quad & \frac{\partial \mathcal{L}}{\partial p_o} = \frac{(1-\beta^2)\lambda - \beta - 2p_o + 2\beta p_f + r + 1}{1 - \beta^2} = 0, \\ & \frac{\partial \mathcal{L}}{\partial p_f} = \frac{\beta^3 \lambda - \beta(\lambda - 2p_o + r + 1) - 2p_f + 1}{1 - \beta^2} = 0, \\ & \hat{s} - q_o \geq 0, \\ & \lambda \geq 0, \\ & \lambda(\hat{s} - q_o) = 0. \end{aligned}$$

By solving the above problem, the restaurant's optimal online and offline channel prices are determined as follows:

$$p_o^*, p_f^* = \begin{cases} 1 - \frac{\beta}{2} + (\beta^2 - 1)\hat{s}, \frac{1}{2} & if \quad r \leq \hat{r}, \quad \text{(C.7a)} \\ \frac{r+1}{2}, \frac{1}{2} & if \quad r \geq \hat{r}, \quad \text{(C.7b)} \end{cases}$$

where $\hat{r} = (1-\beta)(1-2\beta\hat{s}-2\hat{s})$. Substituting p_o^* and p_f^* into the restaurant's profit, we have the restaurant's optimal profit as follows:

$$\pi_r^{BFD*} = \begin{cases} \frac{1}{4} - \hat{s}(\beta + r - 1) + (\beta^2 - 1)\hat{s}^2 & if \quad r \leq \hat{r}, \\ \frac{-2\beta + r(2\beta + r - 2) + 2}{4 - 4\beta^2} & if \quad r \geq \hat{r}. \end{cases}$$

(ii) When the supply is smaller than the demand ($\hat{s} \leq q_o$), the restaurant's optimization problem becomes

$$\pi_r^{BFD} = \max_{p_o, p_f} \hat{s}(p_o - r) + p_f q_f,$$
$$s.t. \quad \hat{s} \leq q_o.$$

In this case, the restaurant's profit increases with p_o, as $\frac{\partial \pi_r^{BFD}}{\partial p_o} = \hat{s} + \frac{\beta p_f}{1-\beta^2} > 0$. Therefore, the restaurant will continue to raise p_o until the demand matches the fixed supply $\hat{s}$, at which point the optimal online price is $p_o^* = 1 - \frac{\beta}{2} + (\beta^2 - 1)\hat{s}$. Substituting p_o^* into the restaurant's profit, we obtain the optimal offline channel price $p_f^* = \frac{1}{2}$. Consequently, we have the restaurant's optimal profit $\pi_r^{BFD*} = \frac{1}{4} - \hat{s}(\beta + r - 1) + (\beta^2 - 1)\hat{s}^2$.

Combining the Case (i) and Case (ii), we can show that Case (ii) is dominated by Case (i) since

$$\pi_r^{BFD*}|_{\hat{s} \geq q_o} - \pi_r^{BFD*}|_{\hat{s} \leq q_o} = \begin{cases} 0 & if \quad r \leq \hat{r}, \\ \frac{(\beta + r - 2(\beta^2 - 1)\hat{s} - 1)^2}{4(1-\beta^2)} > 0 & if \quad r \geq \hat{r}. \end{cases}$$

Hence, the restaurant's best responses are listed in Equation (C.7). Next, we consider the platform's optimization problem in the following two cases.

(a) Anticipating $p_o^* = 1 - \frac{\beta}{2} + (\beta^2 - 1)\hat{s}$, $p_f^* = \frac{1}{2}$, then the platform's profit in Equation (C.2) becomes

$$\pi_p^{BFD} = \hat{s}(r - c),$$
$$s.t. \quad r \leq \hat{r},$$

which increases with r. Hence, the platform increases the commission until $r^* = \hat{r}$, resulting $\pi_p^{BFD*} = \hat{s}((\beta - 1)(2(\beta + 1)\hat{s} - 1) - c)$.

(b) Anticipating $p_o^* = \frac{r+1}{2}$, $p_f^* = \frac{1}{2}$, then the platform's profit in Equation (C.2) becomes

$$\pi_p^{BFD} = \frac{(c-r)(\beta + r - 1)}{2(1-\beta^2)},$$
$$s.t. \quad r \geq \hat{r},$$

which is concave in r since $\frac{\partial^2 \pi_p^{BFD}}{\partial r^2} = -\frac{1}{1-\beta^2} < 0$. Hence, the platform profit is maximized at $\tilde{r} = \frac{1}{2}(1 - \beta + c)$, at which $\frac{\partial \pi_p^{BFD}}{\partial r} = 0$. Additionally, the platform has the constraint $r \geq \hat{r}$. Comparing $\hat{r}$ and $\tilde{r}$, we have the following two subcases:

(b-1) If $\hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}$, then $\tilde{r} \geq \hat{r}$. Hence, the optimal r for the platform is $r^* = \tilde{r}$, resulting $\pi_p^{BFD*} = \frac{(1-\beta-c)^2}{8(1-\beta^2)}$.

(b-2) If $\hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}$, then $\hat{r} \geq \tilde{r}$. Hence the optimal r for the platform is $r^* = \hat{r}$, resulting $\pi_p^{BFD*} = \hat{s}((\beta - 1)(2(\beta + 1)\hat{s} - 1) - c)$.

Combining Case (a) and Case (b), we can show that the platform's profit in Case (a) is dominated by that in Case (b), i.e.,

$$\pi_p^{BFD*}|_{r \geq \hat{r}} - \pi_p^{BFD*}|_{r \leq \hat{r}} = \begin{cases} 0 & if \quad \hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}, \\ \frac{(\beta + c - 4(\beta^2 - 1)\hat{s} - 1)^2}{8(1-\beta^2)} > 0 & if \quad \hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}. \end{cases}$$

Therefore, the equilibrium commission r^* in the BFD contract lies in Case (b). Specifically, we have

$$r^* = \begin{cases} \hat{r} & if \quad \hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}, & \text{(C.8a)} \\ \tilde{r} & if \quad \hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}. & \text{(C.8b)} \end{cases}$$

Consequently, we have the following equilibrium results:

If $\hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}$**,**

$$\begin{array}{ll} p_o^{BFD*} = \frac{2-\beta-2(1-\beta^2)\hat{s}}{2}, & p_f^{BFD*} = \frac{1}{2}, \\ q_o^{BFD*} = \hat{s}, & q_f^{BFD*} = \frac{1}{2} - \beta\hat{s}, \\ \pi_p^{BFD*} = \hat{s}(1-\beta-c+2\hat{s}(\beta^2-1)), & \pi_r^{BFD*} = \frac{4(1-\beta^2)\hat{s}^2+1}{4}, \\ \pi_{sc}^{BFD*} = \frac{4\hat{s}(-\beta-c+1)+4(\beta^2-1)\hat{s}^2+1}{4}. & \end{array} \tag{C.9}$$

If $\hat{s} > \frac{1-\beta-c}{4(1-\beta^2)}$**,**

$$\begin{array}{ll} p_o^{BFD*} = \frac{3-\beta+c}{4}, & p_f^{BFD*} = \frac{1}{2}, \\ q_o^{BFD*} = \frac{-\beta-c+1}{4-4\beta^2}, & q_f^{BFD*} = \frac{\beta(-1-\beta+c)+2}{4(1-\beta^2)}, \\ \pi_p^{BFD*} = \frac{(-\beta-c+1)^2}{8(1-\beta^2)}, & \pi_r^{BFD*} = \frac{-\beta(3\beta+2)+c^2-2(1-\beta)c+5}{16(1-\beta^2)}, \\ \pi_{sc}^{BFD*} = \frac{\beta^2+6\beta(1-c)-3c(c-2)-7}{16(\beta^2-1)}. & \end{array} \tag{C.10}$$

Dynamic-price/dynamic-wage contract (BDD). Using backward induction, we first solve the platform's optimization problem. As before, the supply may either exceed demand or fall short of it.

(i) When supply exceeds demand ($\hat{s} \geq q_o$), we have the following optimization problem for the platform:

$$\begin{aligned} \pi_p^{BDD} &= \max_d q_o(r+d-c), \\ s.t. \quad & \hat{s} \geq q_o = \frac{1}{1+\beta} - \frac{1}{1-\beta^2}(m+r+d) + \frac{\beta}{1-\beta^2}p_f. \end{aligned}$$

The platform's profit is concave in d because $\frac{\partial^2 \pi_p^{BDD}}{\partial d^2} = \frac{2}{\beta^2-1} < 0$. By applying KKT conditions, we can rewrite the above problem as follows:

$$\begin{aligned} \mathcal{L}(d,\lambda) &= q_o(r+d-c) + \lambda(\hat{s}-q_o), \\ s.t. \quad & \frac{\partial \mathcal{L}}{\partial d} = \frac{(\beta^2-1)\lambda+\beta-c+2d+m-\beta p_f+2r-1}{\beta^2-1} = 0, \\ & \hat{s} - q_o \geq 0, \\ & \lambda \geq 0, \\ & \lambda(\hat{s}-q_o) = 0. \end{aligned}$$

By solving this, the platform's optimal delivery fee can be listed as follows:

$$d^* = \begin{cases} 1-m+\beta(p_f-1)-r+(\beta^2-1)\hat{s} & if \quad m \leq \hat{m}, \quad \text{(C.11a)} \\ \frac{1+c-m-2r+\beta(p_f-1)}{2} & if \quad m \geq \hat{m}. \quad \text{(C.11b)} \end{cases}$$

Note that $\hat{m} = 1-c+\beta(p_f-1)+2(\beta^2-1)\hat{s}$. Substituting d^* into the platform's profit, we have the platform's optimal profit as follows:

$$\pi_p^{BDD*} = \begin{cases} -\hat{s}(c+m+\hat{s}-1)+\beta(p_f-1)\hat{s}+\beta^2\hat{s}^2 & if \quad m \leq \hat{m}, \\ \frac{(\beta+c+m-\beta p_f-1)^2}{4(1-\beta^2)} & if \quad m \geq \hat{m}. \end{cases}$$

(ii) When the supply is less than the demand ($\hat{s} \leq q_o$), the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{BDD} &= \max_d \hat{s}(r+d-c), \\ s.t. \quad & \hat{s} \leq q_o = \frac{1}{1+\beta} - \frac{1}{1-\beta^2}(m+r+d) + \frac{\beta}{1-\beta^2}p_f. \end{aligned}$$

In this case, the platform's profit increases with d since $\frac{\partial \pi_p^{BDD}}{\partial d} = \hat{s} > 0$. Hence, the platform always increases d until demand decreases to $\hat{s}$, at which point $d^* = 1-m+\beta(p_f-1)-r+(\beta^2-1)\hat{s}$, leading to $\pi_p^{BDD*} = -\hat{s}(c+m+\hat{s}-1)+\beta(p_f-1)\hat{s}+\beta^2\hat{s}^2$.

Combining Case (i) and (ii) ($\hat{s} \geq q_o$ and $\hat{s} \leq q_o$), we show that Case (ii) is dominated by Case (i) since

$$\pi_p^{BDD*}|_{\hat{s}\geq q_o} - \pi_p^{BDD*}|_{\hat{s}\leq q_o} = \begin{cases} 0 & if \quad m \leq \hat{m}, \\ \frac{(\beta+c+m-\beta p_f-2\beta^2\hat{s}+2\hat{s}-1)^2}{4(1-\beta^2)} & if \quad m \geq \hat{m}. \end{cases}$$

Therefore, the platform's best response is listed in Equation (C.11). Next, we consider the restaurant's optimization problem in the following two cases.

(a) Anticipating the platform's optimal delivery fee $d^* = 1-m+\beta(p_f-1)-r+(\beta^2-1)\hat{s}$, the restaurant's optimization problem in Equation (C.5) becomes

$$\begin{aligned}&\pi_r^{BDD} = \max_{m,p_f} q_o m + p_f q_f = m\hat{s} - p_f(p_f+\beta\hat{s}-1),\\ &s.t. \quad m \leq \hat{m}.\end{aligned}$$

The restaurant's profit increases with m given any p_f. Hence, the restaurant increases the online margin to $m^* = \hat{m}$. Then, substituting $m^* = \hat{m}$ into the restaurant's profit, we can get the optimal offline channel price $p_f^* = \frac{1}{2}$ satisfying $\frac{\partial \pi_r^{BDD}}{\partial p_f} = 0$. Substituting m^* and p_f^* into the restaurant's profit, we can get $\pi_r^{BDD*} = \frac{-4\hat{s}(\beta+c-1)+8(\beta^2-1)\hat{s}^2+1}{4}$.

(b) Anticipating the platform's delivery fee $d^* = \frac{1+c-m-2r+\beta(p_f-1)}{2}$, then the restaurant's optimization problem becomes

$$\begin{aligned}&\pi_r^{BDD} = \max_{m,p_f} q_o m + p_f q_f = \frac{m(\beta+c-2\beta p_f-1)+\beta p_f(\beta-c-\beta p_f+1)+m^2+2(p_f-1)p_f}{2(\beta^2-1)},\\ &s.t. \quad m \geq \hat{m}.\end{aligned}$$

The platform's profit is joint concave in m and p_f because the Hessian matrix is negative definite. Specifically, the Hessian matrix is given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{BDD}}{\partial m^2} & \frac{\partial^2 \pi_r^{BDD}}{\partial m \partial p_f} \\ \frac{\partial^2 \pi_r^{BDD}}{\partial p_f \partial m} & \frac{\partial^2 \pi_r^{BDD}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta^2-1} & -\frac{\beta}{\beta^2-1} \\ -\frac{\beta}{\beta^2-1} & \frac{2-\beta^2}{\beta^2-1} \end{bmatrix}.$$

By applying KKT conditions, we can get the optimal prices

$$m^*, p_f^* = \begin{cases} \frac{-\beta+2(1-c)+4(\beta^2-1)\hat{s}}{2}, \frac{1}{2} & if \quad \hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}, \quad \text{(C.12a)} \\ \frac{1-c}{2}, \frac{1}{2} & if \quad \hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}. \quad \text{(C.12b)} \end{cases}$$

Consequently, the restaurant's optimal profit is $\pi_r^{BDD*} = \frac{-4\hat{s}(\beta+c-1)+8(\beta^2-1)\hat{s}^2+1}{4}$ if $\hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}$; otherwise, the the restaurant's optimal profit is $\pi_r^{BDD*} = \frac{(1-\beta)(\beta+3)+c^2+2(\beta-1)c}{8-8\beta^2}$.

Combining Case (a) and Case (b), we can show that the restaurant's profit in Case (a) is dominated by that in Case (b), i.e.,

$$\pi_r^{BDD*}|_{m\geq\hat{m}} - \pi_r^{BDD*}|_{m\leq\hat{m}} = \begin{cases} 0 & if \quad \hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}, \\ \frac{(\beta+c-4(\beta^2-1)s-1)^2}{8(1-\beta^2)} > 0 & if \quad \hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}. \end{cases}$$

Therefore, the equilibrium prices for the restaurant lie in Equations (C.12).

Next, anticipating the restaurant's optimal decisions, the platform determines the commission fee r in the first stage. However, we find that the platform's profit is independent of the commission fee r, leading to the following equilibrium results:

If $\hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}$**,**

$$\begin{aligned} &m^{BDD*} = \frac{-\beta+2(1-c)+4(\beta^2-1)\hat{s}}{2}, && d^{BDD*} = c-r+\hat{s}(1-\beta^2), \\ &p_o^{BDD*} = \frac{2-\beta-2(1-\beta^2)\hat{s}}{2}, && p_f^{BDD*} = \frac{1}{2}, \\ &q_o^{BDD*} = \hat{s}, && q_f^{BDD*} = \frac{1}{2}-\beta\hat{s}, \\ &\pi_p^{BDD*} = \hat{s}(1-\beta^2), && \pi_r^{BDD*} = \frac{-4\hat{s}(\beta+c-1)+8(\beta^2-1)\hat{s}^2+1}{4}, \\ &\pi_{sc}^{BDD*} = \frac{4\hat{s}(-\beta-c+1)+4(\beta^2-1)\hat{s}^2+1}{4}. \end{aligned} \tag{C.13}$$

If $\hat{s} > \frac{1-\beta-c}{4(1-\beta^2)}$**,**

$$\begin{array}{ll} m^{BDD*} = \frac{2-\beta-2(1-\beta^2)s}{2}, & d^{BDD*} = \frac{-\beta+3c-4r+1}{4}, \\ p_o^{BDD*} = \frac{3-\beta+c}{4}, & p_f^{BDD*} = \frac{1}{2}, \\ q_o^{BDD*} = \frac{-\beta-c+1}{4-4\beta^2}, & q_f^{BDD*} = \frac{\beta(-1-\beta+c)+2}{4(1-\beta^2)}, \\ \pi_p^{BDD*} = \frac{(\beta+c-1)^2}{16(1-\beta^2)}, & \pi_r^{BDD*} = \frac{(1-\beta)(\beta+3)+c^2+2(\beta-1)c}{8-8\beta^2}, \\ \pi_{sc}^{BDD*} = \frac{\beta^2+6\beta(1-c)-3c(c-2)-7}{16(\beta^2-1)}. & \end{array} \tag{C.14}$$

(iii) **Dynamic-price/fixed-wage contract (BDF)**. Since the profit functions and decision sequences for both the platform and the restaurant in this scenario are identical to those in the BDD contract, the equilibrium results are the same as those presented in Equations (C.13) and (C.14).

In summary, comparing equilibrium decisions in three types of contracts (see Equations (C.9), (C.10), (C.13), and (C.14)), we can get our results in Lemma 2. □

C.3. Proof of Lemma 3

Stage 3: platform determines the wage w**, or equivalently, the supply** $s(w)$**.** The total amount of supply is defined as $s = -a + bw$ in Equation (3). This can be rearranged to express w as $w = \frac{a+s}{b}$. To simplify our analysis, we consider the scenario where the platform's decision is the supply s. In this context, the platform has no incentive to choose s such that $s > q_o$ because it could always increase its profit by reducing s (the platform's profit decreases as s increases). Thus, in equilibrium, $s \leq q_o$. Therefore, the platform's optimization problem can be formulated as follows:

$$\begin{aligned} \pi_p^{FD} &= \max_s s(r - \tfrac{a+s}{b}), \\ s.t. \quad s &\leq q_o = \tfrac{1}{1+\beta} - \tfrac{1}{1-\beta^2}(m+r) + \tfrac{\beta}{1-\beta^2} p_f. \end{aligned} \tag{C.15}$$

The platform's profit is concave in s. Next, by applying KKT conditions, we can rewrite the above problem as follows.

$$\begin{aligned} \mathcal{L}(s,\lambda) &= s(r - \tfrac{a+s}{b}) + \lambda(q_o - s), \\ s.t. \quad & \tfrac{\partial \mathcal{L}}{\partial s} = -\tfrac{a+b\gamma-br+2s}{b} = 0, \\ & q_o - s \geq 0, \\ & \lambda \geq 0, \\ & \lambda(q_o - s) = 0. \end{aligned}$$

By solving this, we can get the platform's optimal supply:

$$s^* = \begin{cases} \frac{br-a}{2} & if \quad m \leq \bar{m}, \quad \text{(C.16a)} \\ \frac{1-\beta-m+\beta p_f - r}{1-\beta^2} & if \quad m \geq \bar{m}. \quad \text{(C.16b)} \end{cases}$$

Note that $\bar{m} = \frac{a(1-\beta^2)+r(b(\beta^2-1)-2)+2\beta(p_f-1)+2}{2}$.

Stage 2: restaurant determines online margin m **and the offline channel price** p_f**.** Anticipating the varying optimal supply levels chosen by the platform, the restaurant's pricing decisions will differ accordingly.

(i) Anticipating the platform's optimal supply $s^* = \frac{br-a}{2}$, then the restaurant's optimization problem becomes

$$\begin{aligned} \pi_r^{FD} &= \max_{m,p_f} sm + q_f p_f = \tfrac{br-a}{2} m + q_f p_f, \\ s.t. \quad m &\leq \bar{m} = \tfrac{a(1-\beta^2)+r(b(\beta^2-1)-2)+2\beta(p_f-1)+2}{2}. \end{aligned} \tag{C.17}$$

Taking the derivative of the restaurant's profit with respect to m, we obtain $\frac{\partial \pi_r^{FD}}{\partial m} = \frac{(1-\beta^2)(br-a)+2\beta p_f}{2(1-\beta^2)} > 0$. Hence, given any p_f, the restaurant increases m until $m^* = \bar{m}(p_f)$. Substituting $m^* = \bar{m}(p_f)$ into restaurant's profit, we have $\frac{\partial^2 \pi_r^{FD}}{\partial p_f^2} = \frac{2(1-\beta^2)}{\beta^2-1} < 0$. Thus, the restaurant's optimal p_f satisfying $\frac{\partial \pi_r^{FD}}{\partial p_f} = 0$, which is $p_f^* = \frac{1}{2}$. Hence, the restaurant's optimal prices are $m^* = \bar{m}(p_f^*)$ and $p_f^* = \frac{1}{2}$, respectively, and the restaurant's optimal profit is $\pi_r^{FD*} = \frac{(a-br)(a(\beta^2-1)+r(-b\beta^2+b+2)-2(1-\beta))+1}{4}$.

(ii) Anticipating the platform's optimal supply $s^* = \frac{1-\beta-m+\beta p_f-r}{1-\beta^2}$, then the restaurant's optimization problem becomes

$$\begin{aligned} \pi_r^{FD} &= \max_{m,p_f} sm + q_f p_f = \frac{1-\beta-m+\beta p_f-r}{1-\beta^2} m + q_f p_f, \\ s.t. \quad m &\geq \bar{m} = \frac{a(1-\beta^2)+r(b(\beta^2-1)-2)+2\beta(p_f-1)+2}{2}. \end{aligned} \tag{C.18}$$

The restaurant's profit is jointly concave in m and p_f as the Hessian matrix is negative definite, and the Hessian matrix is given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{FD}}{\partial m^2} & \frac{\partial^2 \pi_r^{FD}}{\partial m \partial p_f} \\ \frac{\partial^2 \pi_r^{FD}}{\partial p_f \partial m} & \frac{\partial^2 \pi_r^{FD}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{2}{\beta^2-1} & \frac{2\beta}{1-\beta^2} \\ \frac{2\beta}{1-\beta^2} & \frac{2}{\beta^2-1} \end{bmatrix}.$$

By applying KKT conditions, the restaurant's optimal prices can be listed as follows:

$$m^*, p_f^* = \begin{cases} \frac{1-r}{2}, \quad \frac{1}{2} & if \quad r \geq \bar{r}, \quad \text{(C.19a)} \\ \frac{2-\beta-a\beta^2+a+r(b(\beta^2-1)-2)}{2}, \quad \frac{1}{2} & if \quad r \leq \bar{r}. \quad \text{(C.19b)} \end{cases}$$

where $\bar{r} = \frac{(\beta-1)(a\beta+a+1)}{b(\beta^2-1)-1}$.

Substituting optimal prices into the restaurant's profit, we can get the corresponding profit of the restaurant is $\pi_r^{FD*} = \frac{r^2-2r(1-\beta)+2(1-\beta)}{4-4\beta^2}$ if $r \geq \bar{r}$; otherwise, $\pi_r^{FD*} = \frac{(a-br)(a(\beta^2-1)+r(-b\beta^2+b+2)+2(\beta-1))+1}{4}$.

Combining Case (i) and Case (ii), we can show that the restaurant's profit in Case (i) is dominated by that in Case (ii) since

$$\pi_r^{FD*}|_{m \geq \bar{m}} - \pi_r^{FD*}|_{m \leq \bar{m}} = \begin{cases} \frac{(\beta^2(a-br)-a+br+\beta+r-1)^2}{4(1-\beta^2)} > 0 & if \quad r \geq \bar{r}, \\ 0 & if \quad r \leq \bar{r}. \end{cases}$$

Therefore, the optimal prices for the restaurant in this stage are given in Equation (C.19).

Stage 1: platform determines the commission fee. We consider the following two cases.

(i) Anticipating the restaurant's optimal channel prices $m^* = \frac{2-\beta-a\beta^2+a+r(b(\beta^2-1)-2)}{2}$, $p_f^* = \frac{1}{2}$, the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{FD} &= \max_r s(r-w) = \frac{(a-br)^2}{4b}, \\ s.t. \quad r &\leq \bar{r}. \end{aligned} \tag{C.20}$$

The platform's profit is convex in r since $\frac{\partial^2 \pi_p^{FD}}{\partial r^2} = \frac{b}{2} > 0$, and the platform's profit gets the minimum value when $r = \frac{a}{b}$. $\bar{r} > \frac{a}{b}$ since $\bar{r} - \frac{a}{b} = \frac{a-b(1-\beta)}{b(b(\beta^2-1)-1)} > 0$. Additionally, the online demand in this stage becomes $s = \frac{-a+br}{2}$. Hence, the commission fee r needs to satisfy $r \geq \frac{a}{b}$ to ensure $s \geq 0$. Therefore, the platform's profit increases in r when $\frac{a}{b} \leq r \leq \bar{r}$, and its optimal commission fee is $r^* = \bar{r}$. Substituting $r^* = \bar{r}$ into the platform's profit, we have $\pi_p^{FD*} = \frac{(a+b(\beta-1))^2}{4b(-b\beta^2+b+1)^2}$.

(ii) Anticipating the restaurant's prices $m^* = \frac{1-r}{2}$, $p_f^* = \frac{1}{2}$, then the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{FD} &= \max_r s(r-w) = \frac{(-\beta-r+1)(2a(\beta^2-1)-2b(\beta^2-1)r+\beta+r-1)}{4b(1-\beta^2)^2}, \\ s.t. \quad r &\geq \bar{r}. \end{aligned} \tag{C.21}$$

The platform's profit is concave in r since $\frac{\partial^2 \pi_p^{FD}}{\partial r^2} = \frac{1}{\beta^2-1} - \frac{1}{2b(\beta^2-1)^2} < 0$. Hence, there exists a

$$r_m = \frac{(1-\beta)(a(-\beta-1)+b(\beta^2-1)-1)}{2b(\beta^2-1)-1}$$

satisfying $\frac{\partial \pi_p^{FD}}{\partial r} = 0$ such that the platform's profit is maximized. $r_m > \bar{r}$ because $r_m - \bar{r} = \frac{b(\beta^2-1)^2(-a+b(1-\beta))}{(b(\beta^2-1)-1)(2b(\beta^2-1)-1)} > 0$. Hence, in this case, the optimal $r^* = r_m$. Substituting r^* into the platform's profit, we have $\pi_p^{FD*} = \frac{(a+b(\beta-1))^2}{4b(2b(1-\beta^2)+1)}$.

Combining Case (i) and Case (ii), the platform's profit in Case (i) is dominated by that in Case (ii) since

$$\pi_p^{FD*}|_{r\geq\bar{r}} - \pi_p^{FD*}|_{r\leq\bar{r}} = \frac{b(\beta^2-1)^2(a+b(\beta-1))^2}{4(-b\beta^2+b+1)^2(2b(1-\beta^2)+1)} > 0.$$

Therefore, the equilibrium commission fee is $r^* = r_m$. Substituting r^* into the optimal decisions in subsequent stages yields the equilibrium results presented in Lemma 3. Furthermore, we have the following equilibrium demands and profits:

$$\begin{aligned} &q_o^{FD*} = s^{FD*} = \tfrac{a-b(1-\beta)}{4b(\beta^2-1)-2}, \quad q_f^{FD*} = \tfrac{-a\beta+b(-\beta-2)(1-\beta)-1}{4b(\beta^2-1)-2}, \\ &\pi_p^{FD*} = -\tfrac{(-a-b\beta+b)^2}{4b(2b(\beta^2-1)-1)}, \\ &\pi_r^{FD*} = \tfrac{a^2(-(\beta^2-1))+2ab(-\beta-1)(1-\beta)^2+b(-\beta-1)(1-\beta)(b(-3\beta-5)(1-\beta)-4)+1}{4(1-2b(\beta^2-1))^2}, \\ &\pi_{sc}^{FD*} = \tfrac{a^2(1-3b(\beta^2-1))+2ab(1-\beta)(3b(\beta^2-1)-1)+b^2(1-\beta)(b(-\beta-7)(-\beta-1)(1-\beta)+3\beta+5)+b}{4b(1-2b(\beta^2-1))^2}. \end{aligned} \tag{C.22}$$

□

C.4. Proof of Lemma 4

Stage 3: platform determines the wage w and the delivery fee d. We first show that $q_o = s$ is the platform's optimal choice in this stage because neither $q_o > s$ nor $q_o < s$ can be optimal. If $q_o > s$, the platform can slightly increase the delivery fee d (which slightly increases the channel price p_o and reduces the demand) such that $\min(q_o, s)$ remains unaltered, thereby increasing the platform's profit. If $q_o < s$, the platform can slightly decrease the wage w (which slightly reduces the supply) such that $\min(q_o, s)$ remains unaltered, thus increasing the platform's profit. Given $q_o = s$, the platform's optimization problem becomes

$$\begin{aligned} &\pi_p^{DD} = \max_{w,d} s(r+d-w), \\ &s.t. \quad s = q_o = \tfrac{1}{1+\beta} - \tfrac{1}{1-\beta^2}(m+r+d) + \tfrac{\beta}{1-\beta^2}p_f. \end{aligned} \tag{C.23}$$

By applying the Lagrange multiplier method, we have

$$\begin{aligned} &\mathcal{L}(w,d,\lambda) = s(r+d-w) + \lambda(q_o - s), \\ &s.t. \quad \tfrac{\partial \mathcal{L}}{\partial w} = 0, \quad \tfrac{\partial \mathcal{L}}{\partial d} = 0, \quad \tfrac{\partial \mathcal{L}}{\partial \lambda} = 0. \end{aligned}$$

By solving the above problem, the platform's optimal wage and delivery fee are listed as follows:

$$w^* = \frac{a(2b(\beta^2-1)-1)+b(m+\beta(1-p_f)-1)}{2b(-b(1-\beta^2)-1)}, \quad d^* = \frac{a(\beta^2-1)-b(\beta^2-1)(\beta+m-\beta p_f+2r-1)+2(\beta+m-\beta p_f+r-1)}{2b(\beta^2-1)-2}.$$

Stage 2: the restaurant sets the online price margin m and the offline channel price p_f. Anticipating the optimal w^* and d^*, the restaurant maximizes the following profit

$$\pi_r^{DD} = ms + q_f p_f = \frac{a(m-\beta p_f)+b(m^2+m(\beta-2\beta p_f-1)+p_f(\beta^2+\beta+\beta^2(-p_f)+2p_f-2))+2(p_f-1)p_f}{2b(\beta^2-1)-2}$$

by setting m and p_f. We find that the restaurant's profit is jointly concave in m and p_f as the Hessian matrix is negative definite. The Hessian matrix is given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{DD}}{\partial m^2} & \frac{\partial^2 \pi_r^{DD}}{\partial m \partial p_f} \\ \frac{\partial^2 \pi_r^{DD}}{\partial p_f \partial m} & \frac{\partial^2 \pi_r^{DD}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{2b}{2b(\beta^2-1)-2} & -\frac{2b\beta}{2b(\beta^2-1)-2} \\ -\frac{2b\beta}{2b(\beta^2-1)-2} & \frac{b(4-2\beta^2)+4}{2b(\beta^2-1)-2} \end{bmatrix}.$$

Hence, the optimal online margin m and the offline channel price p_f satisfy $\frac{\partial \pi_r^{DD}}{\partial m}=0$, $\frac{\partial \pi_r^{DD}}{\partial p_f}=0$, simultaneously, which are characterized by $m^*=\frac{b-a}{2b}$ and $p_f^*=\frac{1}{2}$.

Stage 1: the platform determines the commission fee r. Anticipating the restaurant's optimal m^* and p_f^*, the platform's profit becomes independent of r. Hence, substituting optimal m^* and p_f^* back into d^* and w^*, we can get the equilibrium results in Lemma 4. Furthermore, we have the following equilibrium results:

$$\begin{aligned}
&d^{DD*}=\tfrac{a(3b(1-\beta^2)+2)+b(1-\beta)(b(1-\beta^2)+2)}{4b(b(1-\beta^2)+1)}-r,\\
&q_o^{DD*}=\tfrac{a-b(1-\beta)}{4b(\beta^2-1)-4},\quad q_f^{DD*}=\tfrac{-a\beta+b(-\beta-2)(1-\beta)-2}{4b(\beta^2-1)-4},\\
&\pi_p^{DD*}=\tfrac{(-a-b\beta+b)^2}{16b(b(1-\beta^2)+1)},\quad \pi_r^{DD*}=\tfrac{-a^2+2ab(1-\beta)+b(b(-\beta-3)(1-\beta)-2)}{8b(b(\beta^2-1)-1)},\\
&\pi_{sc}^{DD*}=\tfrac{-3a^2+6ab(1-\beta)+b(b(-\beta-7)(1-\beta)-4)}{16b(b(\beta^2-1)-1)}.
\end{aligned} \tag{C.24}$$

□

C.5. Proof of Lemma 5

Stage 3: platform determines delivery fee d. Similar to the proof of Lemma 3, we equivalently consider the scenario where the platform's decision variable is the supply s in the first stage. Given the fixed s, the platform has no incentive to set the delivery fee d such that $s<q_o$ because it can increase d slightly to raise its profit while keeping $\min(s,q_o)$ unchanged. In equilibrium, this implies that $q_o\le s$. Consequently, the platform's maximization problem can be formulated as follows

$$\begin{aligned}
&\pi_p^{DF}=\max_d q_o(d+r-\tfrac{a+s}{b}),\\
&s.t.\quad q_o=\tfrac{1}{1+\beta}-\tfrac{1}{1-\beta^2}(m+r+d)+\tfrac{\beta}{1-\beta^2}p_f\le s.
\end{aligned} \tag{C.25}$$

The platform's profit is concave in d since $\frac{\partial^2 \pi_p^{DF}}{\partial d^2}=\frac{2}{\beta^2-1}<0$. By applying KKT conditions, we can rewrite the above problem as follows:

$$\begin{aligned}
&\mathcal{L}(d,\lambda)=q_o(d+r-\tfrac{a+s}{b})+\lambda(s-q_o),\\
&s.t.\quad \tfrac{\partial \mathcal{L}}{\partial d}=-\tfrac{a-b(\beta+2d+m-\beta p_f+2r-1)+s}{b(\beta^2-1)}=0,\\
&\qquad s-q_o\ge 0,\\
&\qquad \lambda\ge 0,\\
&\qquad \lambda(s-q_o)=0.
\end{aligned}$$

By solving this, the platform's optimal delivery fee can be listed as follows:

$$d^*=\begin{cases}1-m+\beta(p_f-1)-r+(\beta^2-1)s & if\quad m\le\tilde{m}, \quad (C.26a)\\ \frac{a-b(\beta+m-\beta p_f+2r-1)+s}{2b} & if\quad m\ge\tilde{m}. \quad (C.26b)\end{cases}$$

Note that $\tilde{m}=\frac{-a+b\beta(p_f-1)+2b(\beta^2-1)s+b-s}{b}$.

Stage 2: the restaurant sets the online price margin m and the offline channel price p_f.

(i) Anticipating the platform's delivery fee $d^*=1-m+\beta(p_f-1)-r+(\beta^2-1)s$, then the restaurant's optimization problem becomes

$$\begin{aligned}
&\pi_r^{DF}=\max_{m,p_f} q_o m+q_f p_f=ms-p_f(p_f+\beta s-1),\\
&s.t.\quad m\le\tilde{m}.
\end{aligned} \tag{C.27}$$

Taking the derivative of the restaurant's profit with respect to m, we obtain $\frac{\partial \pi_r^{DF}}{\partial m}=s>0$. Hence, given any p_f, the restaurant increases the margin until $m=\tilde{m}$. Substituting $m=\tilde{m}$ into restaurant's profit, we have $\frac{\partial^2 \pi_r^{DF}}{\partial p_f^2}=-2<0$. Thus, the restaurant's optimal p_f satisfying $\frac{\partial \pi_r^{DF}}{\partial p_f}=0$, which is $p_f=\frac{1}{2}$. Therefore, the restaurant's optimal prices are $m^* = \frac{-2a+b(-\beta+4(\beta^2-1)s+2)-2s}{2b}$ and $p_f^* = \frac{1}{2}$. In this case, the restaurant's profit is $\pi_r^{DF*} = \frac{-4s(a+s)+4(1-\beta)bs(1-2(\beta+1)s)+b}{4b}$.

(ii) Anticipating the platform's delivery fee $d^* = \frac{a-b(\beta+m-\beta p_f+2r-1)+s}{2b}$, then the restaurant's optimization problem becomes

$$\begin{aligned} \pi_r^{DF} &= \max_{m,p_f} q_o m + q_f p_f = \frac{a(m-\beta p_f)+b(m^2+m(\beta-2\beta p_f-1)+\beta p_f(\beta-\beta p_f+1)+2(p_f-1)p_f)+s(m-\beta p_f)}{2b(\beta^2-1)}, \\ s.t.\quad & m \geq \tilde{m}. \end{aligned} \tag{C.28}$$

We find that the restaurant's profit is jointly concave in m and p_f as the Hessian matrix is negative definite, and the Hessian matrix is given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{DF}}{\partial m^2} & \frac{\partial^2 \pi_r^{DF}}{\partial m \partial p_f} \\ \frac{\partial^2 \pi_r^{DF}}{\partial p_f \partial m} & \frac{\partial^2 \pi_r^{DF}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta^2-1} & \frac{\beta}{1-\beta^2} \\ \frac{\beta}{1-\beta^2} & \frac{2-\beta^2}{\beta^2-1} \end{bmatrix}.$$

By applying KKT conditions, the restaurant's optimal prices can be listed as follows:

$$m^*, p_f^* = \begin{cases} \frac{-a+b-s}{2b}, \quad \frac{1}{2} & if \quad s \geq \bar{s}, & \text{(C.29a)} \\ \frac{-2a+b(-\beta+4(\beta^2-1)s+2)-2s}{2b}, \quad \frac{1}{2} & if \quad s \leq \bar{s}, & \text{(C.29b)} \end{cases}$$

where $\bar{s} = \frac{a+b(\beta-1)}{4b(\beta^2-1)-1}$.

Substituting optimal prices into the restaurant's profit, we can get the corresponding profit of the restaurant is $\pi_r^{DF*} = -\frac{a^2+2a(b(\beta-1)+s)-b^2(\beta-1)(\beta+3)+2b(\beta-1)s+s^2}{8b^2(\beta^2-1)}$ if $s \geq \bar{s}$; otherwise, $\pi_r^{DF*} = \frac{-4s(a+s)+4(\beta-1)bs(2(\beta+1)s-1)+b}{4b}$.

Combining Case (i) and Case (ii), we can show that the restaurant's profit in Case (i) is dominated by that in Case (ii) since

$$\pi_r^{DF*}|_{m\geq\tilde{m}} - \pi_r^{DF*}|_{m\leq\tilde{m}} = \begin{cases} \frac{(a+b(\beta-4\beta^2 s+4s-1)+s)^2}{8b^2(1-\beta^2)} > 0 & if \quad s \geq \bar{s}, \\ 0 & if \quad s \leq \bar{s}. \end{cases}$$

Hence, the restaurant's optimal prices at this stage are given in Equation (C.29).

Stage 1: platform determines the supply s. We consider the following two cases.

(i) Anticipating the restaurant's optimal prices $m^* = \frac{-a+b-s}{2b}$, $p_f^* = \frac{1}{2}$, the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{DF} &= \max_s q_o(d + r - \tfrac{a+s}{b}) = \frac{(a+b(\beta-1)+s)^2}{16b^2(1-\beta^2)}, \\ s.t.\quad & s \geq \bar{s}. \end{aligned} \tag{C.30}$$

The platform's profit is convex in s since $\frac{\partial^2 \pi_p^{DF}}{\partial s^2} = \frac{1}{8b^2(1-\beta^2)} > 0$, and the platform's profit is minimized when $s = -a + b(1-\beta)$. We can show that $-a + b(1-\beta) - \bar{s} = \frac{4b(\beta^2-1)(a+b(\beta-1))}{4b(1-\beta^2)+1} > 0$. Additionally, the online demand in this stage becomes $q_o = \frac{a+b(\beta-1)+s}{4b(\beta^2-1)}$, and it decreases with s since $\frac{\partial q_o}{\partial s} = \frac{1}{4b(\beta^2-1)} < 0$, and $q_o = 0$ when $s = -a+b(1-\beta)$. Hence, s needs to satisfy $s \leq -a+b(1-\beta)$ to ensure the positive demand. Therefore, the platform's profit decreases in s when $\bar{s} \leq s \leq -a + b(1-\beta)$, and its optimal supply is $s^* = \bar{s}$.

(ii) Anticipating the restaurant's prices $m^* = \frac{-2a+b(-\beta+4(\beta^2-1)s+2)-2s}{2b}$, $p_f^* = \frac{1}{2}$, then the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{DF} &= \max_s q_o(d + r - \tfrac{a+s}{b}) = s^2(1-\beta^2), \\ s.t.\quad & s \leq \bar{s}. \end{aligned} \tag{C.31}$$

The platform's profit is convex increasing in s, so the platform's optimal supply is $s^* = \bar{s}$.

Combining these two cases, we can get the platform's optimal $s^* = \bar{s}$. Then, substituting s^* into the m^*, p_f^*, and d^*, we can get the equilibrium results in Lemma 5. Furthermore, we have the following equilibrium results:

$$\begin{aligned}
&m^{DF*} = \tfrac{4\beta^2(a-b)-4a+\beta+4b}{2-8b(\beta^2-1)}, \quad d^{DF*} = \tfrac{(1-\beta)(3a(\beta+1)-b\beta^2+b+1)}{4b(1-\beta^2)+1} - r,\\
&q_o^{DF*} = \tfrac{a-b(1-\beta)}{4b(\beta^2-1)-1}, \quad q_f^{DF*} = \tfrac{-2a\beta+2b(-\beta-2)(1-\beta)-1}{8b(\beta^2-1)-2},\\
&\pi_p^{DF*} = -\tfrac{(\beta^2-1)(-a-b\beta+b)^2}{(1-4b(\beta^2-1))^2},\\
&\pi_r^{DF*} = \tfrac{-8a^2(\beta^2-1)+16ab(-\beta-1)(1-\beta)^2+8b(\beta^2-1)(b(-\beta-3)(1-\beta)-1)+1}{4(1-4b(\beta^2-1))^2},\\
&\pi_{sc}^{DF*} = \tfrac{-12a^2(\beta^2-1)+24ab(-\beta-1)(1-\beta)^2+4b(\beta^2-1)(b(-\beta-7)(1-\beta)-2)+1}{4(1-4b(\beta^2-1))^2}.
\end{aligned} \tag{C.32}$$

□

C.6. Proof of Proposition 1

First, we compare the online channel price under the FD contract with its benchmark (BFD). To make a fair comparison, we substitute $c = w^{FD*}$ (see Equation (20)) into the equilibrium online channel price in the BFD case (see Equation (12)), we can get

$$p_o^{BFD*} = \begin{cases} \frac{2-\beta-2\hat{s}(1-\beta^2)}{2} & if \quad \hat{s} \le \frac{(4b(1-\beta^2)+1)(b(1-\beta)-a)}{8b(1-\beta^2)(2b(1-\beta^2)+1)}, \\ \frac{a(4b(\beta^2-1)-1)+b(-4b(\beta-3)(\beta-1)(\beta+1)+3\beta-7)}{8b(2b(\beta^2-1)-1)} & if \quad \hat{s} \ge \frac{(4b(1-\beta^2)+1)(b(1-\beta)-a)}{8b(1-\beta^2)(2b(1-\beta^2)+1)}. \end{cases}$$

Then we compare p_o^{FD*} with p_o^{BFD*}. If $\hat{s} \le \frac{(4b(1-\beta^2)+1)(b(1-\beta)-a)}{8b(1-\beta^2)(2b(1-\beta^2)+1)}$,

$$p_o^{FD*} - p_o^{BFD*} = \frac{1}{2} + \frac{(1-\beta)(1+a(\beta+1)+b(1-\beta^2))}{2+4b(1-\beta^2)} - \frac{2-\beta-2\hat{s}(1-\beta^2)}{2} = \frac{(1-\beta^2)(a+b(\beta-4\beta^2\hat{s}+4\hat{s}-1)+2\hat{s})}{4b(1-\beta^2)+2},$$

which is smaller than 0 when $\hat{s} \le \frac{a+b(\beta-1)}{4b(\beta^2-1)-2} = q_o^{FD*}$ (see Equation (C.22)); otherwise, $p_o^{FD*} - p_o^{BFD*} \ge 0$ when $\hat{s} \ge q_o^{FD*}$. If $\hat{s} \ge \frac{(4b(1-\beta^2)+1)(b(1-\beta)-a)}{8b(1-\beta^2)(2b(1-\beta^2)+1)}$,

$$p_o^{FD*} - p_o^{BFD*} = \frac{1}{2} + \frac{(1-\beta)(1+a(\beta+1)+b(1-\beta^2))}{2+4b(1-\beta^2)} - \frac{a(4b(\beta^2-1)-1)+b(-4b(\beta-3)(\beta-1)(\beta+1)+3\beta-7)}{8b(2b(\beta^2-1)-1)} = \frac{a+b(\beta-1)}{8b(2b(\beta^2-1)-1)} > 0.$$

Since we can show that $\frac{(4b(1-\beta^2)+1)(b(1-\beta)-a)}{8b(1-\beta^2)(2b(1-\beta^2)+1)} - q_o^{FD*} = \frac{b(1-\beta)-a}{8b(1-\beta^2)(2b(1-\beta^2)+1)} > 0$, we have $p_o^{FD*} < p_o^{BFD*}$ if $\hat{s} < q_o^{FD*}$.

Second, we compare the online channel price under the DD contract with its benchmark (BDD). Similarly, we substitute $c = w^{DD*}$ (see Equation (26)) into the equilibrium online channel price in the BDD case (see Equation (12)), we can get

$$p_o^{BDD*} = \begin{cases} \frac{2-\beta-2\hat{s}(1-\beta^2)}{2} & if \quad \hat{s} \le \frac{(4b(1-\beta^2)+3)(b(1-\beta)-a)}{16b(1-\beta^2)(b(1-\beta^2)+1)}, \\ \frac{1}{4}(3-\beta+\frac{a+b(\beta-1)}{4b(b(\beta^2-1)-1)}+\frac{a}{b}) & if \quad \hat{s} \ge \frac{(4b(1-\beta^2)+3)(b(1-\beta)-a)}{16b(1-\beta^2)(b(1-\beta^2)+1)}. \end{cases}$$

Then we compare p_o^{DD*} with p_o^{BDD*}. If $\hat{s} \le \frac{(4b(1-\beta^2)+3)(b(1-\beta)-a)}{16b(1-\beta^2)(b(1-\beta^2)+1)}$,

$$p_o^{DD*} - p_o^{BDD*} = \frac{(1-\beta^2)(a+b(3-\beta))+2(2-\beta)}{4b(1-\beta^2)+4} - \frac{2-\beta-2\hat{s}(1-\beta^2)}{2} = \frac{(1-\beta^2)(a+b(\beta-4\beta^2\hat{s}+4\hat{s}-1)+4\hat{s})}{4+4b(1-\beta^2)},$$

which is smaller than 0 when $\hat{s} < \frac{a+b\beta-b}{4(b\beta^2-b-1)} = q_o^{DD*}$ (see Equation (C.24)); otherwise, $p_o^{DD*} - p_o^{BDD*} \ge 0$ when $\hat{s} \ge q_o^{DD*}$. If $\hat{s} \ge \frac{(4b(1-\beta^2)+3)(b(1-\beta)-a)}{16b(1-\beta^2)(b(1-\beta^2)+1)}$,

$$p_o^{DD*} - p_o^{BDD*} = \frac{(1-\beta^2)(a+b(3-\beta))+2(2-\beta)}{4b(1-\beta^2)+4} - \frac{1}{4}(3-\beta+\frac{a+b(\beta-1)}{4b(b(\beta^2-1)-1)}+\frac{a}{b}) = \frac{3(a+b(\beta-1))}{16b(b(\beta^2-1)-1)} > 0.$$

Since we can show that $\frac{(4b(1-\beta^2)+3)(b(1-\beta)-a)}{16b(1-\beta^2)(b(1-\beta^2)+1)} - q_o^{DD*} = -\frac{3(a+b(\beta-1))}{16b(\beta^2-1)(b(\beta^2-1)-1)} > 0$, we have $p_o^{DD*} < p_o^{BDD*}$ if $\hat{s} < q_o^{DD*}$.

Third, we compare the online channel price under the DF contract with its benchmark (BDF). Similarly, we substitute $c = w^{DF*}$ (see Equation (32)) into the equilibrium online channel price in the BDF case (see Equation (12)), we can get

$$p_o^{BDF*} = \begin{cases} \frac{2-\beta-2\hat{s}(1-\beta^2)}{2} & if \quad \hat{s} \le \frac{a+b(\beta-1)}{4b(\beta^2-1)-1}, \\ \frac{1}{4}(3-\beta+\frac{(\beta-1)(4a(\beta+1)+1)}{4b(\beta^2-1)-1}) & if \quad \hat{s} \ge \frac{a+b(\beta-1)}{4b(\beta^2-1)-1}. \end{cases}$$

Then we compare p_o^{DF*} with p_o^{BDF*}. If $\hat{s} \le \frac{a+b(\beta-1)}{4b(\beta^2-1)-1}$,

$$p_o^{DF*} - p_o^{BDF*} = \frac{2a(1-\beta^2)+2b(3-\beta)(1-\beta^2)+2-\beta}{8b(1-\beta^2)+2} - \frac{2-\beta-2\hat{s}(1-\beta^2)}{2} = \frac{(1-\beta^2)(a+b(\beta-4\beta^2\hat{s}+4\hat{s}-1)+\hat{s})}{4b(1-\beta^2)+1},$$

which is smaller than 0 when $\hat{s} < \frac{a+b(\beta-1)}{4b(\beta^2-1)-1} = q_o^{DF*}$ (see Equation (C.32)); otherwise, $p_o^{DF*} - p_o^{BDF*} \ge 0$ when $\hat{s} \ge q_o^{DF*}$. If $\hat{s} \ge \frac{a+b(\beta-1)}{4b(\beta^2-1)-1}$,

$$p_o^{DF*} - p_o^{BDF*} = 0.$$

Since $\frac{a+b(\beta-1)}{4b(\beta^2-1)-1} - q_o^{DF*} = 0$, we have $p_o^{DF*} < p_o^{BDF*}$ if $\hat{s} < q_o^{DF*}$.

Combining all three cases, we have our results in Proposition 1. □

C.7. Proof of Proposition 2

(i) Based on Equations (18), (24), and (30), we can derive

$$p_o^{DD*} - p_o^{FD*} = \frac{(1-\beta^2)(b(1-\beta)-a)}{4(b(1-\beta^2)+1)(2b(1-\beta^2)+1)} > 0; \quad p_o^{FD*} - p_o^{DF*} = \frac{(1-\beta^2)(b(1-\beta)-a)}{2(2b(1-\beta^2)+1)(4b(1-\beta^2)+1)} > 0.$$

(ii) Based on Equations (C.22), (C.24), and (C.32), we can derive

$$q_o^{DF*} - q_o^{FD*} = \frac{-a+b(1-\beta)}{2(2b(1-\beta^2)+1)(4b(1-\beta^2)+1)} > 0; \quad q_o^{FD*} - q_o^{DD*} = \frac{-a+b(1-\beta)}{4(b(1-\beta^2)+1)(2b(1-\beta^2)+1)} > 0.$$

(iii) Based on Equations (20), (26), and (32), we can derive

$$w^{DF*} - w^{FD*} = \frac{b(1-\beta)-a}{2b(2b(1-\beta^2)+1)(4b(1-\beta^2)+1)} > 0; \quad w^{FD*} - w^{DD*} = \frac{b(1-\beta)-a}{4b(b(1-\beta^2)+1)(2b(1-\beta^2)+1)} > 0.$$

□

C.8. Proof of Proposition 3

Take the partial derivatives of p_o^i, w^i, q_o^i, π_p^i, and π_r^i ($i \in \{FD, DD, DF\}$, defined in Equations (18), (24), (30), (20), (26), (32), (C.22), (C.24), (C.32)) with respect to a, b, and β, respectively, we can get the results in Proposition 3. We omit the detailed information here; please contact the authors for further details. □

C.9. Proof of Proposition 4

Based on Equations (C.22), (C.24), and (C.32), we can derive

(i) Platform's profit:

$$\pi_p^{DF*} - \pi_p^{DD*} = -\frac{(1-8b(1-\beta^2))(a+b(\beta-1))^2}{16b(1-4b(\beta^2-1))^2(1+b(1-\beta^2))}.$$

Hence, $\pi_p^{DF*} - \pi_p^{DD*} > 0$ if $b > \frac{1}{8(1-\beta^2)}$; otherwise, $\pi_p^{DF*} - \pi_p^{DD*} \le 0$. Additionally, we have

$$\pi_p^{FD*} - \pi_p^{DF*} = \frac{(1+4b(1-\beta^2)(1+2b(1-\beta^2)))(a+b(\beta-1))^2}{4b(1-4b(\beta^2-1))^2(2b(1-\beta^2)+1)} > 0;$$
$$\pi_p^{FD*} - \pi_p^{DD*} = \frac{(2b(1-\beta^2)+3)(a+b(\beta-1))^2}{16b(b(1-\beta^2)+1)(2b(1-\beta^2)+1)} > 0.$$

(ii) Restaurant's profit:

$$\pi_r^{DF*} - \pi_r^{DD*} = -\frac{(1-8b(1-\beta^2))(a+b(\beta-1))^2}{8b(1-4b(\beta^2-1))^2(1+b(1-\beta^2))}.$$

Hence, $\pi_r^{DF*} - \pi_r^{DD*} > 0$ if $b > \frac{1}{8(1-\beta^2)}$; otherwise, $\pi_r^{DF*} - \pi_r^{DD*} \leq 0$. Additionally, we have

$$\pi_r^{DD*} - \pi_r^{FD*} = \frac{(2b(1-\beta^2+b(\beta^2-1)^2)+1)(a+b(\beta-1))^2}{8b(1-2b(\beta^2-1))^2(b(1-\beta^2)+1)} > 0;$$
$$\pi_r^{DF*} - \pi_r^{FD*} = \frac{(1-\beta^2)(8b(1-\beta^2)(2b(1-\beta^2)+3)+7)(a+b(\beta-1))^2}{4(4b(1-\beta^2)+1)^2(2b(1-\beta^2)+1)^2} > 0.$$

(iii) Supply chain profit:

$$\pi_{sc}^{DF*} - \pi_{sc}^{FD*} = \frac{(8b^2(1-\beta^2)^2-b\beta^2+b-1)(a+b(\beta-1))^2}{4b(4b(1-\beta^2)+1)^2(2b(1-\beta^2)+1)^2}.$$

Hence, $\pi_{sc}^{DF*} - \pi_{sc}^{FD*} > 0$ when $8b^2(1-\beta^2)^2 - b\beta^2 + b - 1 > 0$, that is, $b > \frac{\sqrt{33}-1}{16(1-\beta^2))}$; otherwise, $\pi_{sc}^{DF*} - \pi_{sc}^{FD*} \leq 0$. Additionally, we have

$$\pi_{sc}^{DF*} - \pi_{sc}^{DD*} = \frac{3(8b(\beta^2-1)+1)(a+b(\beta-1))^2}{16b(1-4b(\beta^2-1))^2(b(\beta^2-1)-1)}.$$

Hence, $\pi_{sc}^{DF*} - \pi_{sc}^{DD*} > 0$ if $b > \frac{1}{8(1-\beta^2)}$; otherwise, $\pi_{sc}^{DF*} - \pi_{sc}^{DD*} \leq 0$. Combining

$$\pi_{sc}^{FD*} - \pi_{sc}^{DD*} = \frac{(4b(1-\beta^2)+1)(a+b(\beta-1))^2}{16b(1-2b(\beta^2-1))^2(b(1-\beta^2)+1)} > 0,$$

we have our results in Proposition 4 (iii). □

C.10. Proof of Proposition 5

(i) Based on Equations (C.1) , (C.22), (C.24), and (C.32), we can derive

$$q_o^{DF*} + q_f^{DF*} - (q_o^{FD*} + q_f^{FD*}) = \frac{(1-\beta)(b(1-\beta)-a)}{2(2b(1-\beta^2)+1)(4b(1-\beta^2)+1)} > 0;$$
$$q_o^{FD*} + q_f^{FD*} - (q_o^{DD*} + q_f^{DD*}) = \frac{(1-\beta)(b(1-\beta)-a)}{4(b(1-\beta^2)+1)(2b(1-\beta^2)+1)} > 0.$$

(ii) Similarly, we can derive

$$q_o^{DF*} - q_o^{C*} = \frac{(2b(\beta^2-1)+1)(b(1-\beta)-a)}{2(b(1-\beta^2)+1)(4b(1-\beta^2)+1)};$$
$$q_o^{DF*} + q_f^{DF*} - (q_o^{C*} + q_f^{C*}) = \frac{(1-\beta)(2b(\beta^2-1)+1)(b(1-\beta)-a)}{2(b(1-\beta^2)+1)(4b(1-\beta^2)+1)}.$$

Hence, $q_o^{DF*} > q_o^{C*}$ and $q_o^{DF*} + q_f^{DF*} > q_o^{C*} + q_f^{C*}$ when $1 - 2b(1-\beta^2) > 0$. □

C.11. Proof of Proposition 6

Based on Equations (B.2) and (B.4), we can derive

$$CS^{DF*} - CS^{FD*} = \frac{(1-\beta^2)(8b(1-\beta^2)+3)(a+b(\beta-1))^2}{8(1-4b(\beta^2-1))^2(1-2b(\beta^2-1))^2} > 0;$$
$$CS^{FD*} - CS^{DD*} = \frac{(1-\beta^2)(4b(1-\beta^2)+3)(a+b(\beta-1))^2}{32(1-2b(\beta^2-1))^2(b(\beta^2-1)-1)^2} > 0;$$
$$DS^{DF*} - DS^{FD*} = \frac{(8b(1-\beta^2)+3)(a+b(\beta-1))^2}{8b(1-4b(\beta^2-1))^2(1-2b(\beta^2-1))^2} > 0;$$
$$DS^{FD*} - DS^{DD*} = \frac{(4b(1-\beta^2)+3)(a+b(\beta-1))^2}{32b(1-2b(\beta^2-1))^2(b(\beta^2-1)-1)^2} > 0.$$

Additionally, based on Equations (B.5), (B.2), (B.4), (C.22), (C.24), and (C.32), we can derive

$$SW^{DF*} - SW^{FD*} = \frac{(b(1-\beta)(\beta+1)(24b(1-\beta^2)+13)+1)(a+b(\beta-1))^2}{8b(1-4b(\beta^2-1))^2(1-2b(\beta^2-1))^2} > 0;$$
$$SW^{FD*} - SW^{DD*} = \frac{(12b(\beta^2-1)-5)(a+b(\beta-1))^2}{32b(1-2b(\beta^2-1))^2(b(\beta^2-1)-1)} > 0.$$

□

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Appendix D: Additional Analytical Results of the Wage Rate Model

ASSUMPTION D.1. *When the platform determines the wage rate, the model parameters satisfy* $1-\beta-y\phi \geq 0$.

This assumption ensures that online and dine-in demands are positive regardless of the contracting schemes. If not, the platform may close the online channel. In the following, we will briefly list the platform and restaurant's optimization problem in each type of contract and then characterize the equilibrium solutions respectively.

Fixed-Price/Dynamic-Wage Contract In the first stage, the platform chooses the commission fee r to maximize the profit:

$$\max_{r} \pi_p = \min(s(w), q_o)(r - wq_o), \tag{D.1}$$

where w is the wage rate offered by the platform, and wq_o is the driver's wage.

In the second stage, given r, the restaurant sets the online sales margin and the offline channel price to maximize the following profit function,

$$\max_{m,p_f} \pi_r = \min(s(w), q_o)m + q_f p_f. \tag{D.2}$$

Note that the online channel price is given by $p_o = r + m$.

Finally, in the last stage of the game, given the commission fee r and channel prices in two channels (p_o, p_f), the platform maximizes its profit by solving for optimal wage rate,

$$\max_{w} \pi_p = \min(s(w), q_o)(r - wq_o). \tag{D.3}$$

Solving backward, we characterize the equilibrium outcomes in the next lemma.

LEMMA D1. *When the platform determines the wage rate, then under the FD contract, the equilibrium commission fee, the equilibrium online and offline channel prices, and the equilibrium wage rate are*

$$r^{FD*} = \frac{(1-\beta)((\beta+1)N(1-\beta+y\phi)+1-\phi^2)}{2(1-\beta^2)N-\phi^2+1}, \tag{D.4}$$

$$p_o^{FD*} = \frac{(2-\beta)(1-\phi^2)+(1-\beta^2)N(-\beta+y\phi+3)}{2(2(1-\beta^2)N-\phi^2+1)}, \tag{D.5}$$

$$p_f^{FD*} = \frac{1}{2}, \tag{D.6}$$

$$w^{FD*} = \frac{1-(1-\beta)\phi^2-\beta+y\phi(4(1-\beta^2)N+1)-y\phi^3}{N(1-\beta-y\phi)}. \tag{D.7}$$

Dynamic-Price/Dynamic-Wage Contract In the first stage, the platform chooses the commission fee r to maximize its profit:

$$\max_{r} \pi_p = \min(s(w), q_o)(r + d - wq_o). \tag{D.8}$$

In the second stage, the restaurant decides the dine-in channel price alongside the online sales margin to maximize its profit, given by

$$\max_{m,p_f} \pi_r = \min(s(w), q_o)m + q_f p_f. \tag{D.9}$$

Finally, in the last stage of the game, the platform sets the delivery fee, d, and the wage rate, w, to maximize its profit:

$$\max_{w,d} \pi_p = \min(s(w), q_o)(r + d - wq_o). \tag{D.10}$$

Note that the online channel price is given by $p_o = r + m + d$. We employ backward induction to solve for the equilibrium outcomes, as outlined in Lemma D2.

LEMMA D2. *When the platform determines the wage rate, then under the DD contract, the equilibrium online and offline channel prices and the equilibrium wage rate are*

$$p_o^{DD*} = \frac{2(2-\beta)(1-\phi^2)+(1-\beta^2)N(-\beta+y\phi+3)}{4((1-\beta^2)N-\phi^2+1)}, \tag{D.11}$$
$$p_f^{DD*} = \frac{1}{2}, \tag{D.12}$$
$$w^{DD*} = \frac{-(1-\beta)\phi^2-\beta+y\phi(4(1-\beta^2)N+3)-3y\phi^3+1}{N(-\beta-y\phi+1)}. \tag{D.13}$$

Dynamic-Price/Fixed-Wage Contract Under the DF contract, the platform first commits to the wage rate paid to the delivery drivers and asks for a commission from the restaurant. We can write the platform's problem in the first stage of the game as

$$\max_{w,r} \pi_p = \min(s(w), q_o)(r + d - wq_o). \tag{D.14}$$

In the second stage, the restaurant sets its margin on online orders alongside the dine-in channel prices to maximize its profit, given by

$$\max_{m,p_f} \pi_r = \min(s(w), q_o)m + q_f p_f. \tag{D.15}$$

Finally, the platform sets the online channel price $p_o = r + d + m$ by announcing its delivery fee d in the third stage to maximize its profit

$$\max_{d} \pi_p = \min(s(w), q_o)(r + d - wq_o). \tag{D.16}$$

The following lemma characterizes the equilibrium outcome.

LEMMA D3. *When the platform determines the wage rate, then under the DF contract, the equilibrium online and offline channel prices and the equilibrium wage rate are*

$$p_o^{DF*} = -\frac{(\beta-2)(\phi^2-1)+2(\beta^2-1)N(\beta-y\phi-3)}{2(4(\beta^2-1)N+\phi^2-1)}, \tag{D.17}$$
$$p_f^{DF*} = \frac{1}{2}, \tag{D.18}$$
$$w^{DF*} = \frac{(\beta-1)(4(\beta+1)Ny\phi-\phi^2+1)}{N(\beta+y\phi-1)}. \tag{D.19}$$

Next, we will derive the optimal driver's surplus. Recall that the driver's utility is defined as

$$U = \max_{l_p,l_o} l_p w q_o + l_o y - \frac{1}{2}l_p^2 - \frac{1}{2}l_o^2 - \phi l_p l_o.$$

Solving the driver's optimization problem, we have

$$l_p^* = \frac{wq_o-\phi y}{1-\phi^2}; \quad l_o^* = \frac{y-wq_o\phi}{1-\phi^2}.$$

Substituting the optimal amount of labor l_p^* and l_o^* into the driver's utility, we can get

$$U = \frac{q_o^2w^2-2q_oyw\phi+y^2}{2-2\phi^2}, \tag{D.20}$$

which demonstrates one driver's optimal utility. Given the system contains N amount of drivers, the driver's surplus in this scenario is equivalent to the driver's utility multiplied by the number of drivers in the system, i.e., $DS^i = U^i N$, $i \in \{FD, DD, DF\}$. Substituting optimal wage rate w^{i*} into the driver's surplus, we can get

$$\begin{aligned}
DS^{FD*} &= \frac{N(-(\beta-1)^2(\phi^2-1)+y^2(\phi^2(16(\beta^2-1)N-7)+4(1-2(\beta^2-1)N)^2+3\phi^4)-2(\beta-1)y\phi(\phi^2-1))}{8(2(\beta^2-1)N+\phi^2-1)^2}, \\
DS^{DD*} &= \frac{N(-(\beta-1)^2(\phi^2-1)+y^2(\phi^2(32(\beta^2-1)N-31)+16(-\beta^2N+N+1)^2+15\phi^4)-2(\beta-1)y\phi(\phi^2-1))}{32((\beta^2-1)N+\phi^2-1)^2}, \\
DS^{DF*} &= \frac{N(-(\beta-1)^2(\phi^2-1)+y^2(\phi^2(8(\beta^2-1)N-1)+(1-4(\beta^2-1)N)^2)-2(\beta-1)y\phi(\phi^2-1))}{2(4(\beta^2-1)N+\phi^2-1)^2}.
\end{aligned} \tag{D.21}$$

For customers, since we adopt the same demand functions as the main model, the customer's surplus is the same as Equation (B.3). Substituting optimal channel prices and demands p_o^{i*}, p_f^{i*}, q_o^{i*}, q_f^{i*} into Equation (B.3), we can get

$$\begin{aligned}
CS^{FD*} &= \frac{(\beta^2-1)N^2(3\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-5)+4(\beta^2-1)N(\phi^2-1)+\phi^4-2\phi^2+1}{8(2(\beta^2-1)N+\phi^2-1)^2}, \\
CS^{DD*} &= \frac{(\beta^2-1)N^2(3\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-5)+8(\beta^2-1)N(\phi^2-1)+4(\phi^2-1)^2}{32((\beta^2-1)N+\phi^2-1)^2}, \\
CS^{DF*} &= \frac{4(\beta^2-1)N^2(3\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-5)+8(\beta^2-1)N(\phi^2-1)+\phi^4-2\phi^2+1}{8(4(\beta^2-1)N+\phi^2-1)^2}.
\end{aligned} \tag{D.22}$$

Appendix E: Additional Analytical Results of the Alternative Demand Model

Following the same analysis in Section 4, we can get the equilibrium solutions in the following Lemmas for each type of contract. The solving process is similar to the main part and the wage rate model, so we omit it here. Similar to the previous two cases, we make the following assumption to ensure the positive online demand.

Assumption E.1. *Consider the demand functions in* (35) *and* (36), *the model parameters satisfy* $-a(1+\beta)+b \geq 0$.

Lemma E1. *Consider the demand functions in* (35) *and* (36), *then under the FD contract, the equilibrium channel prices, driver's wage, demands, and profits are*

$$\begin{aligned}
p_o^{FD*} &= \frac{1}{4}\left(\frac{2a+1}{2b+\beta+1} + \frac{1-\alpha}{2\beta+1} + \alpha + \frac{1}{\beta+1} + 1\right), \\
p_f^{FD*} &= \frac{\alpha\beta+\alpha+\beta}{4\beta+2}, \\
w^{FD*} &= \frac{4ab+a\beta+a+b}{4b^2+2\beta b+2b}, \\
q_o^{FD*} &= \frac{b-a(\beta+1)}{2(2b+\beta+1)}, \\
q_f^{FD*} &= \frac{1}{2}\left(\frac{\beta(a\beta+a+b+\beta+1)}{(\beta+1)(2b+\beta+1)} + \alpha\right), \\
\pi_p^{FD*} &= \frac{(a\beta+a-b)^2}{4b(\beta+1)(2b+\beta+1)}, \\
\pi_r^{FD*} &= \frac{1}{16}\left(\frac{4a^2-1}{2b+\beta+1} - \frac{2(2a+1)^2 b}{(2b+\beta+1)^2} + \frac{2(\alpha-1)^2}{2\beta+1} + 2(\alpha+1)^2 - \frac{3}{\beta+1}\right), \\
\pi_{sc}^{FD*} &= \frac{1}{16}\left(\frac{4a^2(\beta+1)(3b+\beta+1)-8ab(3b+\beta+1)-b(8b+3\beta+3)}{b(2b+\beta+1)^2} + \frac{4(\alpha\beta+\alpha+\beta)^2+6\beta+3}{(\beta+1)(2\beta+1)}\right).
\end{aligned} \tag{E.1}$$

Lemma E2. *Consider the demand functions in* (35) *and* (36), *then under the DD contract, the equilibrium channel prices, driver's wage, demands, and profits are*

$$\begin{aligned}
p_o^{DD*} &= \frac{1}{4}\left(\frac{a+1}{b+\beta+1} + \frac{1-\alpha}{2\beta+1} + \alpha + \frac{1}{\beta+1} + 1\right), \\
p_f^{DD*} &= \frac{\alpha\beta+\alpha+\beta}{4\beta+2}, \\
w^{DD*} &= \frac{1}{4}\left(\frac{a+1}{b+\beta+1} + \frac{3a}{b}\right), \\
q_o^{DD*} &= \frac{b-a(\beta+1)}{4(b+\beta+1)}, \\
q_f^{DD*} &= \frac{1}{4}\left(\frac{\beta((a+2)\beta+a+b+2)}{(\beta+1)(b+\beta+1)} + 2\alpha\right), \\
\pi_p^{DD*} &= \frac{(a\beta+a-b)^2}{16b(\beta+1)(b+\beta+1)}, \\
\pi_r^{DD*} &= \frac{1}{8}\left(\frac{a^2}{b} - \frac{(a+1)^2}{b+\beta+1} + \frac{(\alpha-1)^2}{2\beta+1} + (\alpha+1)^2 - \frac{1}{\beta+1}\right), \\
\pi_{sc}^{DD*} &= \frac{1}{16}\left(\frac{3a^2}{b} - \frac{3(a+1)^2}{b+\beta+1} + \frac{2(\alpha-1)^2}{2\beta+1} + 2(\alpha+1)^2 - \frac{1}{\beta+1}\right).
\end{aligned} \tag{E.2}$$

LEMMA E3. *Consider the demand functions in* (35) *and* (36)*, then under the DF contract, the equilibrium channel prices, driver's wage, demands, and profits are*

$$
\begin{aligned}
p_o^{DF*} &= \tfrac{1}{4}\big(\tfrac{4a+1}{4b+\beta+1} + \tfrac{1-\alpha}{2\beta+1} + \alpha + \tfrac{1}{\beta+1} + 1\big),\\
p_f^{DF*} &= \tfrac{\alpha\beta+\alpha+\beta}{4\beta+2},\\
w^{DF*} &= \tfrac{4a+1}{4b+\beta+1},\\
q_o^{DF*} &= \tfrac{b-a(\beta+1)}{4b+\beta+1},\\
q_f^{DF*} &= \tfrac{1}{2}\big(\tfrac{\beta(2a(\beta+1)+2b+\beta+1)}{(\beta+1)(4b+\beta+1)} + \alpha\big),\\
\pi_p^{DF*} &= \tfrac{(a\beta+a-b)^2}{(\beta+1)(4b+\beta+1)^2},\\
\pi_r^{DF*} &= \tfrac{1}{8}\big(\tfrac{16a^2-1}{4b+\beta+1} - \tfrac{4(4a+1)^2 b}{(4b+\beta+1)^2} + \tfrac{(\alpha-1)^2}{2\beta+1} + (\alpha+1)^2 - \tfrac{1}{\beta+1}\big),\\
\pi_{sc}^{DF*} &= \tfrac{1}{16}\big(\tfrac{3(4a+1)(4a(\beta+1)-8b-\beta-1)}{(4b+\beta+1)^2} + \tfrac{4\alpha((\alpha+2)\beta+\alpha)}{2\beta+1} - \tfrac{1}{\beta+1} + \tfrac{2}{2\beta+1} + 2\big).
\end{aligned}
\tag{E.3}
$$

Appendix F: Additional Tables

In this section, we demonstrate the partial derivatives of p_o^i, w^i, q_o^i, π_p^i, and π_r^i ($i \in \{FD, DD, DF\}$, defined in Equations (18), (24), (30), (20), (26), (32), (C.22), (C.24), (C.32)) with respect to a, b, and β, respectively.

Table F.1: Partial Derivatives of p_o^i, w^i, q_o^i, π_p^i and π_r^i, $i \in \{DD, FD\}$ with respect to a, b and β

Derivatives	$i = DD$	$i = FD$
$\frac{\partial p_o^{i*}}{\partial a}$	$\frac{1-\beta^2}{4b(1-\beta^2)+4} > 0$	$\frac{1-\beta^2}{4b(1-\beta^2)+2} > 0$
$\frac{\partial p_o^{i*}}{\partial b}$	$-\frac{(\beta-1)^2(\beta+1)(a\beta+a+1)}{4(-b\beta^2+b+1)^2} < 0$	$-\frac{(1-\beta)^2(\beta+1)(2a(\beta+1)+1)}{2(1-2b(1-\beta^2))^2} < 0$
$\frac{\partial p_o^{i*}}{\partial \beta}$	$\frac{-2(a\beta+1)-b^2(1-\beta^2)^2-b(1-\beta)(\beta+3)}{4(-b\beta^2+b+1)^2} < 0$	$\frac{-2a\beta-2b^2(1-\beta^2)^2+b(\beta-1)(\beta+3)-1}{2(1-2b(1-\beta^2))^2} < 0$
$\frac{\partial w^{i*}}{\partial a}$	$\frac{4b(1-\beta^2)+3}{4b(b(1-\beta^2)+1)} > 0$	$\frac{4b(1-\beta^2)+1}{2b(2b(1-\beta^2)+1)} > 0$
$\frac{\partial w^{i*}}{\partial b}$	$-\frac{a(2b(1-\beta^2)(2b(1-\beta^2)+3)+3)+b^2(\beta+1)(1-\beta)^2}{4b^2(b(1-\beta^2)+1)^2} < 0$	$-\frac{4ab(1-\beta^2)(2b(1-\beta^2)+1)+a+2b^2(\beta+1)(1-\beta)^2}{2b^2(1-2b(1-\beta^2))^2} < 0$
$\frac{\partial w^{i*}}{\partial \beta}$	$-\frac{2a\beta+b(1-\beta)^2+1}{4(b(1-\beta^2)+1)^2} < 0$	$-\frac{4a\beta+2b(1-\beta)^2+1}{2(1-2b(1-\beta^2))^2} < 0$
$\frac{\partial q_o^{i*}}{\partial a}$	$\frac{1}{4b(\beta^2-1)-4} < 0$	$\frac{1}{4b(\beta^2-1)-2} < 0$
$\frac{\partial q_o^{i*}}{\partial b}$	$\frac{(1-\beta)(a\beta+a+1)}{4(-b\beta^2+b+1)^2} > 0$	$\frac{(1-\beta)(2a(\beta+1)+1)}{2(1-2b(\beta^2-1))^2} > 0$
$\frac{\partial q_o^{i*}}{\partial \beta}$	$-\frac{b(2a\beta+b(1-\beta)^2+1)}{4(-b\beta^2+b+1)^2} < 0$	$-\frac{b(4a\beta+2b(\beta-1)^2+1)}{2(1-2b(\beta^2-1))^2} < 0$
$\frac{\partial \pi_p^{i*}}{\partial a}$	$\frac{a+b(\beta-1)}{8b(-b\beta^2+b+1)} < 0$	$\frac{a+b\beta-b}{-4\beta^2b^2+4b^2+2b} < 0$
$\frac{\partial \pi_p^{i*}}{\partial b}$	$\frac{(a+b(\beta-1))(a(2b(\beta^2-1)-1)+b(\beta-1))}{16b^2(-b\beta^2+b+1)^2} > 0$	$\frac{(a+b(\beta-1))(a(4b(\beta^2-1)-1)+b(\beta-1))}{4b^2(1-2b(\beta^2-1))^2} > 0$
$\frac{\partial \pi_p^{i*}}{\partial \beta}$	$\frac{(a+b(\beta-1))(a\beta-\beta b+b+1)}{8(-b\beta^2+b+1)^2} < 0$	$\frac{(a+b(\beta-1))(2a\beta-2b(\beta-1)+1)}{2(1-2b(\beta^2-1))^2} < 0$
$\frac{\partial \pi_r^{i*}}{\partial a}$	$\frac{a+b(\beta-1)}{4b(-b\beta^2+b+1)} < 0$	$-\frac{(\beta^2-1)(a+b(\beta-1))}{2(1-2b(\beta^2-1))^2} < 0$
$\frac{\partial \pi_r^{i*}}{\partial b}$	$\frac{(a+b(\beta-1))(a(2b(\beta^2-1)-1)+b(\beta-1))}{8b^2(-b\beta^2+b+1)^2} > 0$	$\frac{(\beta-1)^2(\beta+1)(2a(\beta+1)+1)(a+b(\beta-1))}{2(2b(\beta^2-1)-1)^3} > 0$
$\frac{\partial \pi_r^{i*}}{\partial \beta}$	$\frac{(a+b(\beta-1))(a\beta-\beta b+b+1)}{4(-b\beta^2+b+1)^2} < 0$	$\frac{(a+b(\beta-1))(a\beta-b(\beta-1)(-2\beta(a\beta+a+1)+2b(\beta^2-1)-1))}{2(2b(\beta^2-1)-1)^3} < 0$

Appendix G: Proofs of Results in Extensions

G.1. Proof of Lemma D1

Stage 3: platform determines the wage rate w, or equivalently, the supply $s(w)$. The total amount of supply is defined as $s = \frac{wq_o - \phi y}{1-\phi^2}N$ in Equation (34). Equivalently, we can get $w = \frac{Ny\phi + s(1-\phi^2)}{q_o N}$. Next, we consider the scenario where the platform's decision is the supply s for tractability. Similar to the FD contract in the base model, the platform chooses a s such that in equilibrium, $s \le q_o$. Hence, the platform's maximization problem can be written as

$$\begin{aligned} \pi_p^{FD} &= \max_s s\left(r - \frac{Ny\phi + s(1-\phi^2)}{N}\right), \\ s.t. \quad & s \le q_o = \frac{1}{1+\beta} - \frac{1}{1-\beta^2}(m+r) + \frac{\beta}{1-\beta^2}p_f. \end{aligned} \tag{G.1}$$

It is obvious that the platform's profit is concave in s. By applying KKT conditions, the platform's optimal supply can be listed as follows:

$$s^* = \begin{cases} \frac{N(r-y\phi)}{2(1-\phi^2)} & if \quad m \le \bar{m}, \quad \text{(G.2a)} \\ \frac{1-\beta-m+\beta p_f - r}{1-\beta^2} & if \quad m \ge \bar{m}. \quad \text{(G.2b)} \end{cases}$$

Table F.2: Partial Derivatives of p_o^{DF}, w^{DF}, q_o^{DF}, π_p^{DF} and π_r^{DF}, with respect to a, b and β

Derivatives	$i = DF$
$\frac{\partial p_o^{DF*}}{\partial a}$	$\frac{1-\beta^2}{4b(1-\beta^2)+1} > 0$
$\frac{\partial p_o^{DF*}}{\partial b}$	$-\frac{(1-\beta)^2(\beta+1)(4a(\beta+1)+1)}{(1-4b(1-\beta^2))^2} < 0$
$\frac{\partial p_o^{DF*}}{\partial \beta}$	$-\frac{4a\beta+8b^2(1-\beta^2)^2+2b(1-\beta)(\beta+3)+1}{2(1-4b(\beta^2-1))^2} < 0$
$\frac{\partial w^{DF*}}{\partial a}$	$\frac{4(1-\beta^2)}{4b(1-\beta^2)+1} > 0$
$\frac{\partial w^{DF*}}{\partial b}$	$-\frac{4(1-\beta)^2(\beta+1)(4a(\beta+1)+1)}{(4b(1-\beta^2)+1)^2} < 0$
$\frac{\partial w^{DF*}}{\partial \beta}$	$-\frac{8a\beta+4b(1-\beta)^2+1}{(4b(1-\beta^2)+1)^2} < 0$
$\frac{\partial q_o^{DF*}}{\partial a}$	$\frac{1}{4b(\beta^2-1)-1} < 0$
$\frac{\partial q_o^{DF*}}{\partial b}$	$\frac{(1-\beta)(4a(\beta+1)+1)}{(1-4b(\beta^2-1))^2} > 0$
$\frac{\partial q_o^{DF*}}{\partial \beta}$	$-\frac{b(8a\beta+4b(\beta-1)^2+1)}{(1-4b(\beta^2-1))^2} < 0$
$\frac{\partial \pi_p^{DF*}}{\partial a}$	$-\frac{2(\beta^2-1)(a+b(\beta-1))}{(1-4b(\beta^2-1))^2} < 0$
$\frac{\partial \pi_p^{DF*}}{\partial b}$	$\frac{2(\beta-1)^2(\beta+1)(4a(\beta+1)+1)(a+b(\beta-1))}{(4b(\beta^2-1)-1)^3} > 0$
$\frac{\partial \pi_p^{DF*}}{\partial \beta}$	$\frac{2(a+b(\beta-1))(a\beta-b(\beta-1)(-2(2a(\beta+1)\beta+\beta)+4b(\beta^2-1)-1))}{(4b(\beta^2-1)-1)^3} < 0$
$\frac{\partial \pi_r^{DF*}}{\partial a}$	$-\frac{4(\beta^2-1)(a+b(\beta-1))}{(1-4b(\beta^2-1))^2} < 0$
$\frac{\partial \pi_r^{DF*}}{\partial b}$	$\frac{4(\beta-1)^2(\beta+1)(4a(\beta+1)+1)(a+b(\beta-1))}{(4b(\beta^2-1)-1)^3} > 0$
$\frac{\partial \pi_r^{DF*}}{\partial \beta}$	$\frac{4(a+b(\beta-1))(a\beta-b(\beta-1)(-2(2a(\beta+1)\beta+\beta)+4b(\beta^2-1)-1))}{(4b(\beta^2-1)-1)^3} < 0$

Note that $\bar{m} = \frac{-r(N(1-\beta^2)+2)+(1-\beta^2)Ny\phi-2\beta(1-p_f)+2\phi^2(\beta-\beta p_f+r-1)+2}{2(1-\phi^2)}$.

Stage 2: restaurant determines online margin m and the offline channel price p_f.

(i) Anticipating the platform's optimal supply $s^* = \frac{N(r-y\phi)}{2(1-\phi^2)}$, then the restaurant's optimization problem becomes

$$\begin{aligned} \pi_r^{FD} &= \max_{m,p_f} sm + q_f p_f = \frac{N(r-y\phi)}{2(1-\phi^2)} m + q_f p_f, \\ s.t. \quad & m \le \bar{m}. \end{aligned} \tag{G.3}$$

Taking the derivative of the restaurant's profit with respect to m, we obtain

$$\frac{\partial \pi_r^{FD}}{\partial m} = \frac{N(r-y\phi)}{2-2\phi^2} + \frac{\beta p_f}{1-\beta^2} > 0$$

as long as $s > 0$ ($r > y\phi$). Thus, given any p_f, the restaurant increases m until $m = \bar{m}$. Substituting $m = \bar{m}$ into restaurant's profit, we have $\frac{\partial^2 \pi_r^{FD}}{\partial p_f^2} = -2 < 0$. Hence, the restaurant's optimal p_f satisfying $\frac{\partial \pi_r^{FD}}{\partial p_f} = 0$, which is $p_f^* = \frac{1}{2}$. Therefore, the restaurant's optimal prices are

$$m^* = \bar{m}(p_f^*), \quad p_f^* = \frac{1}{2},$$

respectively, and the restaurant's optimal profit is

$$\pi_r^{FD*} = \frac{1}{4}\left(\frac{(\beta^2-1)N^2(r-y\phi)^2}{(\phi^2-1)^2} + \frac{2N(-\beta-r+1)(r-y\phi)}{1-\phi^2} + 1\right).$$

(ii) Anticipating the platform's optimal supply $s^* = \frac{1-\beta-m+\beta p_f-r}{1-\beta^2}$, then the restaurant's optimization problem becomes

$$\begin{aligned} \pi_r^{FD} &= \max_{m,p_f} sm + q_f p_f = \frac{1-\beta-m+\beta p_f-r}{1-\beta^2} m + q_f p_f, \\ s.t. \quad & m \ge \bar{m}. \end{aligned} \tag{G.4}$$

The restaurant's profit is jointly concave in m and p_f as the Hessian matrix is negative definite. Specifically, the Hessian matrix is given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{FD}}{\partial m^2} & \frac{\partial^2 \pi_r^{FD}}{\partial m \partial p_f} \\ \frac{\partial^2 \pi_r^{FD}}{\partial p_f \partial m} & \frac{\partial^2 \pi_r^{FD}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{2}{\beta^2-1} & \frac{2\beta}{1-\beta^2} \\ \frac{2\beta}{1-\beta^2} & \frac{2}{\beta^2-1} \end{bmatrix}.$$

By applying KKT conditions, the restaurant's optimal prices can be listed as follows:

$$m^*, p_f^* = \begin{cases} \frac{1-r}{2}, \quad \frac{1}{2} & if \quad r \geq \bar{r}, \\ \frac{\beta+r(-\beta^2 N+N+2)+(\beta^2-1)Ny\phi+\phi^2(-(\beta+2r-2))-2}{2(\phi^2-1)}, \quad \frac{1}{2} & if \quad r \leq \bar{r}, \end{cases}$$

where $\bar{r} = \frac{(\beta-1)((\beta+1)Ny\phi-\phi^2+1)}{(\beta^2-1)N+\phi^2-1}$. Substituting optimal prices into the restaurant's profit, we can get the corresponding profit of the restaurant is $\pi_r^{FD*} = \frac{-2\beta+r(2\beta+r-2)+2}{4-4\beta^2}$ if $r \geq \bar{r}$; otherwise, $\pi_r^{FD*} = \frac{1}{4}(\frac{(\beta^2-1)N^2(r-y\phi)^2}{(\phi^2-1)^2} + \frac{2N(-\beta-r+1)(r-y\phi)}{1-\phi^2} + 1)$.

Combining Case (i) and Case (ii), we can show that the restaurant's profit in Case (i) is dominated by that in Case (ii) since

$$\pi_r^{FD*}|_{m \geq \bar{m}} - \pi_r^{FD*}|_{m \leq \bar{m}} = \begin{cases} -\frac{(\beta-\beta^2 Nr+Nr+(\beta^2-1)Ny\phi-\phi^2(\beta+r-1)+r-1)^2}{4(\beta^2-1)(\phi^2-1)^2} > 0 & if \quad r \geq \bar{r}, \\ 0 & if \quad r \leq \bar{r}. \end{cases}$$

Thus the optimal prices for the restaurant in this stage are:

$$m^*, p_f^* = \begin{cases} \frac{1-r}{2}, \quad \frac{1}{2} & if \quad r \geq \bar{r}, \\ \frac{\beta+r(-\beta^2 N+N+2)+(\beta^2-1)Ny\phi+\phi^2(-(\beta+2r-2))-2}{2(\phi^2-1)}, \quad \frac{1}{2} & if \quad r \leq \bar{r}, \end{cases}$$

where $\bar{r} = \frac{(\beta-1)((\beta+1)Ny\phi-\phi^2+1)}{(\beta^2-1)N+\phi^2-1}$.

Stage 1: platform determines the commission fee. We consider the following two cases.

(i) Anticipating the restaurant's optimal channel prices $m^* = \frac{\beta+r(-\beta^2 N+N+2)+(\beta^2-1)Ny\phi+\phi^2(-(\beta+2r-2))-2}{2(\phi^2-1)}$, $p_f^* = \frac{1}{2}$, the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{FD} &= \max_r s(r - wq_o) = \frac{N(r-y\phi)^2}{4(1-\phi^2)}, \\ s.t. &\quad r \leq \bar{r}. \end{aligned} \tag{G.5}$$

The platform's profit is convex in r since $\frac{\partial^2 \pi_p^{FD}}{\partial r^2} = \frac{N}{2(1-\phi^2)} > 0$, and the platform's profit gets the minimum value when $r = y\phi$. $\bar{r} > y\phi$ since $\bar{r} - y\phi = \frac{(1-\phi^2)(1-\beta-y\phi)}{(1-\beta^2)N+1-\phi^2} > 0$. Additionally, the online demand in this stage becomes $s = \frac{N(r-y\phi)}{2-2\phi^2}$. Hence, the commission fee r needs to satisfy $r \geq y\phi$ to ensure $s \geq 0$. Therefore, the platform's profit increases in r when $y\phi \leq r \leq \bar{r}$, and its optimal commission fee is $r^* = \bar{r}$. Substituting r^* into the platform's profit, we have $\pi_p^{FD*} = \frac{N(1-\phi^2)(1-\beta-y\phi)^2}{4((1-\beta^2)N+1-\phi^2)^2}$.

(ii) Anticipating the restaurant's prices $m^* = \frac{1-r}{2}$, $p_f^* = \frac{1}{2}$, then the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{FD} &= \max_r s(r - wq_o) = \frac{(1-\beta-r)(r(2(1-\beta^2)N-\phi^2+1)+(\beta-1)(2(\beta+1)Ny\phi-\phi^2+1))}{4(1-\beta^2)^2 N}, \\ s.t. &\quad r \geq \bar{r}. \end{aligned} \tag{G.6}$$

The platform's profit is concave in r since $\frac{\partial^2 \pi_p^{FD}}{\partial r^2} = -\frac{2(1-\beta^2)N+(1-\phi^2)}{2(\beta^2-1)^2 N} < 0$. Hence, there exists a

$$r_m = \frac{(1-\beta)((\beta+1)N(-\beta+y\phi+1)-\phi^2+1)}{2(1-\beta^2)N-\phi^2+1}$$

satisfying $\frac{\partial \pi_p^{FD}}{\partial r} = 0$ such that the platform's profit is maximized. $r_m > \bar{r}$ because $r_m - \bar{r} = \frac{(\beta^2-1)^2 N^2(-\beta-y\phi+1)}{((1-\beta^2)N-\phi^2+1)(2(1-\beta^2)N-\phi^2+1)} > 0$. Hence, in this case, the optimal $r^* = r_m$. Substituting r^* into the platform's profit, we have $\pi_p^{FD*} = \frac{N(\beta+y\phi-1)^2}{4(2(1-\beta^2)N-\phi^2+1)}$.

Combining Case (i) and Case (ii), the platform's profit in Case (i) is dominated by that in Case (ii) since

$$\pi_p^{FD*}|_{r\geq\bar{r}} - \pi_p^{FD*}|_{r\leq\bar{r}} = \frac{(\beta^2-1)^2N^3(\beta+y\phi-1)^2}{4((\beta^2-1)N+\phi^2-1)^2(2(1-\beta^2)N+1-\phi^2)} > 0.$$

Therefore, the equilibrium commission fee is $r^* = r_m$. Substituting r^* into the optimal decisions in later stages, we can get the equilibrium results in Lemma D1. Furthermore, we have the following equilibrium results:

$$\begin{aligned} q_o^{FD*} &= s^{FD*} = \frac{N(-\beta-y\phi+1)}{2(2(1-\beta^2)N-\phi^2+1)}, \\ q_f^{FD*} &= \frac{N(-\beta^2-\beta+\beta y\phi+2)-\phi^2+1}{2(2(1-\beta^2)N-\phi^2+1)}, \\ \pi_p^{FD*} &= \frac{N(\beta+y\phi-1)^2}{4(2(1-\beta^2)N-\phi^2+1)}, \\ \pi_r^{FD*} &= \frac{(\beta^2-1)N^2(3\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-5)+4(\beta^2-1)N(\phi^2-1)+\phi^4-2\phi^2+1}{4(2(\beta^2-1)N+\phi^2-1)^2}, \\ \pi_{sc}^{FD*} &= \frac{(\beta^2-1)N^2(\beta^2+\beta(6-6y\phi)-3y\phi(y\phi-2)-7)+N(\phi^2-1)(3\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-5)+(\phi^2-1)^2}{4(2(\beta^2-1)N+\phi^2-1)^2}. \end{aligned} \tag{G.7}$$

□

G.2. Proof of Lemma D2

Stage 3: platform determines the wage rate w and the delivery fee d. Similarly to the proof of Lemma 4, we can show that $q_o = s$ is the platform's optimal choice in this stage. Given $q_o = s$, the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{DD} &= \max_{w,d} s(r+d-wq_o), \\ s.t. \quad s &= q_o = \tfrac{1}{1+\beta} - \tfrac{1}{1-\beta^2}(m+r+d) + \tfrac{\beta}{1-\beta^2}p_f. \end{aligned} \tag{G.8}$$

By applying the Lagrange multiplier method, we have

$$\begin{aligned} \mathcal{L}(w,d,\lambda) &= s(r+d-wq_o) + \lambda(q_o - s), \\ s.t. \quad \tfrac{\partial \mathcal{L}}{\partial w} &= 0, \quad \tfrac{\partial \mathcal{L}}{\partial d} = 0, \quad \tfrac{\partial \mathcal{L}}{\partial \lambda} = 0. \end{aligned} \tag{G.9}$$

By solving the above problem, the platform's optimal wage and delivery fee are listed as follows:

$$\begin{aligned} w^* &= \frac{\beta-\phi^2(\beta+m-\beta p_f-1)+m+y\phi(2(\beta^2-1)N-1)-\beta p_f+y\phi^3-1}{N(\beta+m-\beta p_f+y\phi-1)}, \\ d^* &= \frac{m(-\beta^2N+N-2\phi^2+2)+(\beta^2-1)N(\beta(p_f-1)-2r+y\phi+1)-2(\phi^2-1)(\beta-\beta p_f+r-1)}{2((\beta^2-1)N+\phi^2-1)}. \end{aligned}$$

Stage 2: the restaurant sets the online price margin m and the offline channel price p_f. Anticipating the optimal w^* and d^*, the restaurant maximizes the following profit

$$\pi_r^{DD} = ms + q_f p_f = \frac{m^2N+mN(\beta-2\beta p_f+y\phi-1)+Np_f(\beta^2+\beta-(\beta^2-2)p_f-\beta y\phi-2)-2(p_f-1)p_f(\phi^2-1)}{2((\beta^2-1)N+\phi^2-1)}$$

by setting m and p_f. We find that the restaurant's profit is jointly concave in m and p_f as the Hessian matrix is negative definite, with the Hessian matrix given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{DD}}{\partial m^2} & \frac{\partial^2 \pi_r^{DD}}{\partial m \partial p_f} \\ \frac{\partial^2 \pi_r^{DD}}{\partial p_f \partial m} & \frac{\partial^2 \pi_r^{DD}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{N}{(\beta^2-1)N+\phi^2-1} & -\frac{\beta N}{(\beta^2-1)N+\phi^2-1} \\ -\frac{\beta N}{(\beta^2-1)N+\phi^2-1} & \frac{2(2-\beta^2)N+4(1-\phi^2)}{2((\beta^2-1)N+\phi^2-1)} \end{bmatrix}.$$

Hence, the optimal online margin m and the offline channel price p_f satisfy $\frac{\partial \pi_r^{DD}}{\partial m} = 0$, $\frac{\partial \pi_r^{DD}}{\partial p_f} = 0$, simultaneously, which is characterized in the following two equations:

$$m^* = \tfrac{1-y\phi}{2}; \quad p_f^* = \tfrac{1}{2}.$$

Stage 1: the platform sets the commission fee r. Anticipating the restaurant's optimal m^* and p_f^*, the platform's profit becomes independent of r. Hence, substituting optimal m^* and p_f^* back into d^* and w^*, we can get the equilibrium results in Lemma D2. Furthermore, we have the following equilibrium results:

$$\begin{aligned}
d^{DD*} &= -\frac{(\beta^2-1)N(\beta-3y\phi-1)+2(\phi^2-1)(\beta-y\phi-1)}{4((\beta^2-1)N+\phi^2-1)} - r,\\
q_o^{DD*} &= \frac{N(\beta+y\phi-1)}{4((\beta^2-1)N+\phi^2-1)},\\
q_f^{DD*} &= \frac{N(\beta^2+\beta-\beta y\phi-2)+2\phi^2-2}{4((\beta^2-1)N+\phi^2-1)},\\
\pi_p^{DD*} &= -\frac{N(\beta+y\phi-1)^2}{16((\beta^2-1)N+\phi^2-1)},\\
\pi_r^{DD*} &= \frac{N(\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-3)+2\phi^2-2}{8((\beta^2-1)N+\phi^2-1)},\\
\pi_{sc}^{DD*} &= \frac{(\beta-1)(\beta+7)N+\phi^2(4-3Ny^2)-6(\beta-1)Ny\phi-4}{16((\beta^2-1)N+\phi^2-1)}.
\end{aligned} \tag{G.10}$$

□

G.3. Proof of Lemma D3

Stage 3: platform determines the delivery fee d. The total amount of supply is defined as $s = \frac{wq_o-\phi y}{1-\phi^2}N$ in Equation (34). Equivalently, we can get $w = \frac{Ny\phi+s(1-\phi^2)}{q_oN}$. To simplify our analysis, we consider the scenario where the platform's decision is the supply s in the first stage. Given s, the platform has no incentive to choose delivery fee d such that $s < q_o$ because otherwise, the platform can increase d slightly to increase its profit such that $\min(s, q_o)$ remains unaltered. In other words, in equilibrium, $q_o \le s$. Hence, the platform's maximization problem can be written as

$$\begin{aligned}
\pi_p^{DF} &= \max_d q_o\left(d + r - \frac{Ny\phi+s(1-\phi^2)}{N}\right),\\
s.t. \quad q_o &= \frac{1}{1+\beta} - \frac{1}{1-\beta^2}(m+r+d) + \frac{\beta}{1-\beta^2}p_f \le s.
\end{aligned} \tag{G.11}$$

The platform's profit is concave in the delivery fee since $\frac{\partial \pi_p^{DF}}{\partial d} = \frac{2}{\beta^2-1} < 0$. By applying KKT conditions, the platform's optimal delivery fee can be listed as follows:

$$d^* = \begin{cases} 1 - m + \beta(p_f - 1) - r + (\beta^2-1)s & if \quad m \le \tilde{m}, \quad \text{(G.12a)}\\ \frac{-N(\beta+m-\beta p_f+2r-1)+Ny\phi-s\phi^2+s}{2N} & if \quad m \ge \tilde{m}. \quad \text{(G.12b)} \end{cases}$$

Note that $\tilde{m} = \frac{s(2(\beta^2-1)N+\phi^2-1)}{N} + \beta(p_f-1) - y\phi + 1$.

Stage 2: the restaurant sets the online price margin m and the offline channel price p_f.

(i) Anticipating the platform's delivery fee $d^* = 1 - m + \beta(p_f-1) - r + (\beta^2-1)s$, then the restaurant's optimization problem becomes

$$\begin{aligned}
\pi_r^{DF} &= \max_{m,p_f} q_o m + q_f p_f = ms - p_f(p_f + \beta s - 1),\\
s.t. \quad & m \le \tilde{m}.
\end{aligned} \tag{G.13}$$

Taking the derivative of the restaurant's profit with respect to m, we obtain $\frac{\partial \pi_r^{DF}}{\partial m} = s > 0$. Hence, given any p_f, the restaurant increases the margin until $m = \tilde{m}$. Substituting $m = \tilde{m}$ into restaurant's profit, we have $\frac{\partial^2 \pi_r^{DF}}{\partial p_f^2} = -2 < 0$. Thus, the restaurant's optimal p_f satisfying $\frac{\partial \pi_r^{DF}}{\partial p_f} = 0$, which is $p_f = \frac{1}{2}$. Hence, the restaurant's optimal prices are:

$$m^* = 1 - \frac{\beta}{2} + \frac{s(2(\beta^2-1)N+\phi^2-1)}{N} - y\phi, \quad p_f^* = \frac{1}{2}.$$

In this case, the restaurant's profit is $\pi_r^{DF*} = \frac{s^2(2(\beta^2-1)N+\phi^2-1)}{N} + s(1-\beta-y\phi) + \frac{1}{4}$.

(ii) Anticipating the platform's delivery fee $d^* = \frac{-N(\beta+m-\beta p_f+2r-1)+Ny\phi-s\phi^2+s}{2N}$, then the restaurant's optimization problem becomes

$$\begin{aligned} \pi_r^{DF} &= \max_{m,p_f} q_o m + q_f p_f = \frac{m^2 N + m(N(\beta-2\beta p_f+y\phi-1)-s\phi^2+s)+p_f(N(\beta^2(1-p_t)+\beta+2p_f-\beta y\phi-2)+\beta s(\phi^2-1))}{2(\beta^2-1)N}, \\ s.t. &\quad m \geq \tilde{m}. \end{aligned}$$

We find that the restaurant's profit is jointly concave in m and p_f as the Hessian matrix is negative definite, with the Hessian matrix given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{DF}}{\partial m^2} & \frac{\partial^2 \pi_r^{DF}}{\partial m \partial p_f} \\ \frac{\partial^2 \pi_r^{DF}}{\partial p_f \partial m} & \frac{\partial^2 \pi_r^{DF}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta^2-1} & \frac{\beta}{1-\beta^2} \\ \frac{\beta}{1-\beta^2} & \frac{2-\beta^2}{\beta^2-1} \end{bmatrix}.$$

By applying KKT conditions, the restaurant's optimal prices can be listed as follows:

$$m^*, p_f^* = \begin{cases} \frac{N(1-y\phi)+s(\phi^2-1)}{2N}, \quad \frac{1}{2} & if \quad s \geq \bar{s}, \\ 1-y\phi-\frac{\beta}{2}+\frac{s(2(\beta^2-1)N+\phi^2-1)}{N}, \quad \frac{1}{2} & if \quad s \leq \bar{s}, \end{cases}$$

where $\bar{s} = \frac{N(1-\beta-y\phi)}{4(1-\beta^2)N+1-\phi^2}$.

Substituting optimal prices into the restaurant's profit, we can get the corresponding profit of the restaurant is $\pi_r^{DF*} = \frac{N^2(\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-3)+2Ns(\phi^2-1)(\beta+y\phi-1)-s^2(\phi^2-1)^2}{8(\beta^2-1)N^2}$ if $s \geq \bar{s}$; otherwise, $\pi_r^{DF*} = \frac{s^2(2(\beta^2-1)N+\phi^2-1)}{N} + s(1-\beta-y\phi)+\frac{1}{4}$.

Combining Case (i) and Case (ii), we can show that the restaurant's profit in Case (i) is dominated by that in Case (ii) since

$$\pi_r^{DF*}|_{m\geq\tilde{m}} - \pi_r^{DF*}|_{m\leq\tilde{m}} = \begin{cases} -\frac{(N(\beta-4(\beta^2-1)s+y\phi-1)-s\phi^2+s)^2}{8(\beta^2-1)N^2} > 0 & if \quad s \geq \bar{s}, \\ 0 & if \quad s \leq \bar{s}. \end{cases}$$

Hence, the restaurant's optimal prices at this stage are:

$$m^*, p_f^* = \begin{cases} \frac{N(1-y\phi)+s(\phi^2-1)}{2N}, \quad \frac{1}{2} & if \quad s \geq \bar{s}, & \text{(G.15a)} \\ 1-y\phi-\frac{\beta}{2}+\frac{s(2(\beta^2-1)N+\phi^2-1)}{N}, \quad \frac{1}{2} & if \quad s \leq \bar{s}, & \text{(G.15b)} \end{cases}$$

where $\bar{s} = \frac{N(1-\beta-y\phi)}{4(1-\beta^2)N+1-\phi^2}$.

Stage 1: platform determines the supply s We consider the following two cases.

(i) Anticipating the restaurant's optimal prices $m^* = \frac{N(1-y\phi)+s(\phi^2-1)}{2N}$, $p_f^* = \frac{1}{2}$, the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{DF} &= \max_s q_o(d+r-\tfrac{Ny\phi-s\phi^2+s}{N}) = \frac{(N(\beta+y\phi-1)-s\phi^2+s)^2}{16(1-\beta^2)N^2}, \\ s.t. &\quad s \geq \bar{s}. \end{aligned} \tag{G.16}$$

The platform's profit is convex in s since $\frac{\partial^2 \pi_p^{DF}}{\partial s^2} = \frac{(\phi^2-1)^2}{8(1-\beta^2)N^2} > 0$, and the platform's profit is minimized when $s = \frac{N(\beta+y\phi-1)}{\phi^2-1}$. We can show that $\frac{N(\beta+y\phi-1)}{\phi^2-1} - \bar{s} = \frac{4(\beta^2-1)N^2(\beta+y\phi-1)}{(\phi^2-1)(4(\beta^2-1)N+\phi^2-1)} > 0$. Additionally, the online demand in this stage becomes $q_o = \frac{N(\beta+y\phi-1)-s\phi^2+s}{4(\beta^2-1)N}$, and it decreases with s since $\frac{\partial q_o}{\partial s} = -\frac{\phi^2-1}{4(\beta^2-1)N} < 0$, and $q_o = 0$ when $s = \frac{N(\beta+y\phi-1)}{\phi^2-1}$. Hence, s needs to satisfy $s \leq \frac{N(\beta+y\phi-1)}{\phi^2-1}$ to ensure the positive demand. Therefore, the platform's profit decreases in s when $\bar{s} \leq s \leq \frac{N(\beta+y\phi-1)}{\phi^2-1}$, and its optimal supply is $s^* = \bar{s}$.

(ii) Anticipating the restaurant's prices $m^* = 1-y\phi-\frac{\beta}{2}+\frac{s(2(\beta^2-1)N+\phi^2-1)}{N}$, $p_f^* = \frac{1}{2}$, then the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{DF} &= \max_s q_o(d+r-\tfrac{Ny\phi-s\phi^2+s}{N}) = s^2(1-\beta^2), \\ s.t. &\quad s \leq \bar{s}. \end{aligned} \tag{G.17}$$

The platform's profit is convex increasing in s, so the platform's optimal supply is $s^* = \bar{s}$.

Combining these two cases, we can get the platform's optimal $s^* = \bar{s}$. Then, substituting s^* into the m^*, p_f^*, and d^*, we can get the equilibrium results in Lemma D3. Furthermore, we have the following equilibrium results:

$$\begin{aligned}
m^{DF*} &= \frac{\beta(\phi^2-1)-4(\beta^2-1)N(y\phi-1)}{2(4(\beta^2-1)N+\phi^2-1)}, \\
d^{DF*} &= \frac{(\beta-1)((\beta+1)N(\beta-3y\phi-1)+\phi^2-1)}{4(\beta^2-1)N+\phi^2-1} - r, \\
q_o^{DF*} &= \frac{N(\beta+y\phi-1)}{4(\beta^2-1)N+\phi^2-1}, \\
q_f^{DF*} &= \frac{1}{2} - \frac{\beta N(\beta+y\phi-1)}{4(\beta^2-1)N+\phi^2-1}, \\
\pi_p^{DF*} &= -\frac{(\beta^2-1)N^2(\beta+y\phi-1)^2}{(4(\beta^2-1)N+\phi^2-1)^2}, \\
\pi_r^{DF*} &= \frac{8(\beta^2-1)N^2(\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-3)+8(\beta^2-1)N(\phi^2-1)+\phi^4-2\phi^2+1}{4(4(\beta^2-1)N+\phi^2-1)^2}, \\
\pi_{sc}^{DF*} &= \frac{4(\beta^2-1)N^2(\beta^2+\beta(6-6y\phi)-3y\phi(y\phi-2)-7)+8(\beta^2-1)N(\phi^2-1)+(\phi^2-1)^2}{4(4(\beta^2-1)N+\phi^2-1)^2}.
\end{aligned} \tag{G.18}$$

□

G.4. Proof of Proposition 7

(i) Based on Equations (D.5), (D.11), and (D.17), we can derive

$$p_o^{DD*} - p_o^{FD*} = \frac{(1-\beta^2)N(\phi^2-1)(\beta+y\phi-1)}{4((\beta^2-1)N+\phi^2-1)(2(\beta^2-1)N+\phi^2-1)} > 0; \quad p_o^{FD*} - p_o^{DF*} = \frac{(1-\beta^2)N(\phi^2-1)(\beta+y\phi-1)}{2(2(\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)} > 0.$$

(ii) Based on Equations (D.7), (D.13), (D.19), (G.7), (G.10), and (G.18), we can derive

$$w^{DD*} - w^{FD*} = \frac{2y\phi(\phi^2-1)}{N(\beta+y\phi-1)} > 0; \quad w^{FD*} - w^{DF*} = \frac{y\phi(\phi^2-1)}{N(\beta+y\phi-1)} > 0;$$

$$q_o^{DF*}w^{DF*} - q_o^{FD*}w^{FD*} = -\frac{(\phi^2-1)^2(\beta+y\phi-1)}{2(2(\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)} > 0;$$

$$q_o^{FD*}w^{FD*} - q_o^{DD*}w^{DD*} = -\frac{(\phi^2-1)^2(\beta+y\phi-1)}{4((\beta^2-1)N+\phi^2-1)(2(\beta^2-1)N+\phi^2-1)} > 0.$$

(iii) Based on Equations (G.7), (G.10), and (G.18), we can derive

$$q_o^{DF*} - q_o^{FD*} = \frac{N(\phi^2-1)(\beta+y\phi-1)}{2(2(\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)} > 0; \quad q_o^{FD*} - q_o^{DD*} = \frac{N(\phi^2-1)(\beta+y\phi-1)}{4((\beta^2-1)N+\phi^2-1)(2(\beta^2-1)N+\phi^2-1)} > 0.$$

□

G.5. Proof of Proposition 8

Based on Equations (G.7), (G.10), and (G.18), we can derive

(i) Platform's profit:

$$\pi_p^{DF*} - \pi_p^{DD*} = -\frac{N(\phi^2-1)(8(\beta^2-1)N-\phi^2+1)(\beta+y\phi-1)^2}{16((\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)^2}.$$

Hence, $\pi_p^{DF*} - \pi_p^{DD*} > 0$ if $N > \frac{1-\phi^2}{8-8\beta^2}$; otherwise, $\pi_p^{DF*} - \pi_p^{DD*} \leq 0$. Additionally, we have

$$\begin{aligned}
\pi_p^{FD*} - \pi_p^{DF*} &= -\frac{N(8(\beta^2-1)^2N^2+4(\beta^2-1)N(\phi^2-1)+(\phi^2-1)^2)(\beta+y\phi-1)^2}{4(2(\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)^2} > 0; \\
\pi_p^{FD*} - \pi_p^{DD*} &= -\frac{N(2(\beta^2-1)N+3(\phi^2-1))(\beta+y\phi-1)^2}{16((\beta^2-1)N+\phi^2-1)(2(\beta^2-1)N+\phi^2-1)} > 0.
\end{aligned}$$

(ii) Restaurant's profit:

$$\pi_r^{DF*} - \pi_r^{DD*} = -\frac{N(\phi^2-1)(8(\beta^2-1)N-\phi^2+1)(\beta+y\phi-1)^2}{8((\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)^2}.$$

Hence, $\pi_r^{DF*} - \pi_r^{DD*} > 0$ if $N > \frac{1-\phi^2}{8-8\beta^2}$; otherwise, $\pi_r^{DF*} - \pi_r^{DD*} \leq 0$. Additionally, we have

$$\begin{aligned}
\pi_r^{DD*} - \pi_r^{FD*} &= \frac{N(2(\beta^2-1)^2N^2+2(\beta^2-1)N(\phi^2-1)+(\phi^2-1)^2)(\beta+y\phi-1)^2}{8((\beta^2-1)N+\phi^2-1)(2(\beta^2-1)N+\phi^2-1)^2} > 0; \\
\pi_r^{DF*} - \pi_r^{FD*} &= -\frac{(\beta^2-1)N^2(16(\beta^2-1)^2N^2+24(\beta^2-1)N(\phi^2-1)+7(\phi^2-1)^2)(\beta+y\phi-1)^2}{4(2(\beta^2-1)N+\phi^2-1)^2(4(\beta^2-1)N+\phi^2-1)^2} > 0.
\end{aligned}$$

(iii) Supply chain profit:

$$\pi_{sc}^{DF*} - \pi_{sc}^{FD*} = -\frac{N(\phi^2-1)(8(\beta^2-1)^2N^2+(\beta^2-1)N(\phi^2-1)-(\phi^2-1)^2)(\beta+y\phi-1)^2}{4(2(\beta^2-1)N+\phi^2-1)^2(4(\beta^2-1)N+\phi^2-1)^2}.$$

Hence, $\pi_{sc}^{DF*} - \pi_{sc}^{FD*} > 0$ when $8(\beta^2-1)^2N^2 + (\beta^2-1)N(\phi^2-1) - (\phi^2-1)^2 > 0$, that is, $N > \frac{\sqrt{33}-1}{16(1-\beta^2))}$; otherwise, $\pi_{sc}^{DF*} - \pi_{sc}^{FD*} \leq 0$. Additinally, we have

$$\pi_{sc}^{DF*} - \pi_{sc}^{DD*} = -\frac{3N(\phi^2-1)(8(\beta^2-1)N-\phi^2+1)(\beta+y\phi-1)^2}{16((\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)^2}.$$

Hence, $\pi_{sc}^{DF*} - \pi_{sc}^{DD*} > 0$ if $N > \frac{1-\phi^2}{8-8\beta^2}$; otherwise, $\pi_{sc}^{DF*} - \pi_{sc}^{DD*} \leq 0$. Combining

$$\pi_{sc}^{FD*} - \pi_{sc}^{DD*} = -\frac{N(\phi^2-1)(4(\beta^2-1)N+\phi^2-1)(\beta+y\phi-1)^2}{16((\beta^2-1)N+\phi^2-1)(2(\beta^2-1)N+\phi^2-1)^2} > 0,$$

we have our results in Proposition 8 (iii).

(iv) Customer surplus:

$$CS^{DF*} - CS^{FD*} = -\frac{(\beta^2-1)N^2(\phi^2-1)(8(\beta^2-1)N+3(\phi^2-1))(\beta+y\phi-1)^2}{8(2(\beta^2-1)N+\phi^2-1)^2(4(\beta^2-1)N+\phi^2-1)^2} > 0;$$
$$CS^{FD*} - CS^{DD*} = -\frac{(\beta^2-1)N^2(\phi^2-1)(4(\beta^2-1)N+3(\phi^2-1))(\beta+y\phi-1)^2}{32((\beta^2-1)N+\phi^2-1)^2(2(\beta^2-1)N+\phi^2-1)^2} > 0;$$

(v) Driver surplus:

$$DS^{DF*} - DS^{FD*} = -\frac{N(\phi^2-1)^2(8(\beta^2-1)N+3(\phi^2-1))(\beta+y\phi-1)^2}{8(2(\beta^2-1)N+\phi^2-1)^2(4(\beta^2-1)N+\phi^2-1)^2} > 0;$$
$$DS^{FD*} - DS^{DD*} = -\frac{N(\phi^2-1)^2(4(\beta^2-1)N+3(\phi^2-1))(\beta+y\phi-1)^2}{32((\beta^2-1)N+\phi^2-1)^2(2(\beta^2-1)N+\phi^2-1)^2} > 0.$$

□

G.6. Proof of Proposition B1

Based on Equations (E.1), (E.2), and (E.3), we can derive

(i) Online channel price:

$$p_o^{DD*} - p_o^{FD*} = \frac{b-a(\beta+1)}{4(b+\beta+1)(2b+\beta+1)} > 0; \quad p_o^{FD*} - p_o^{DF*} = \frac{b-a(\beta+1)}{2(2b+\beta+1)(4b+\beta+1)} > 0.$$

(ii) Driver's wage:

$$w^{DF*} - w^{FD*} = -\frac{(\beta+1)(a\beta+a-b)}{2b(2b+\beta+1)(4b+\beta+1)} > 0; \quad w^{FD*} - w^{DD*} = -\frac{(\beta+1)(a\beta+a-b)}{4b(b+\beta+1)(2b+\beta+1)} > 0;$$

(iii) Online demand:

$$q_o^{DF*} - q_o^{FD*} = -\frac{(\beta+1)(a\beta+a-b)}{2(2b+\beta+1)(4b+\beta+1)} > 0;$$

$$q_o^{FD*} - q_o^{DD*} = -\frac{(\beta+1)(a\beta+a-b)}{4(b+\beta+1)(2b+\beta+1)} > 0.$$

(iv) Platform's profit:

$$\pi_p^{DF*} - \pi_p^{DD*} = \frac{(8b-\beta-1)(b-a(\beta+1))^2}{16b(b+\beta+1)(4b+\beta+1)^2}.$$

Hence, $\pi_p^{DF*} - \pi_p^{DD*} > 0$ if $b > \frac{1+\beta}{8}$; otherwise, $\pi_p^{DF*} - \pi_p^{DD*} \leq 0$. Additionally, we have

$$\pi_p^{FD*} - \pi_p^{DF*} = \frac{(8b^2+4(\beta+1)b+(\beta+1)^2)(a\beta+a-b)^2}{4b(\beta+1)(2b+\beta+1)(4b+\beta+1)^2} > 0;$$
$$\pi_p^{FD*} - \pi_p^{DD*} = \frac{(2b+3\beta+3)(b-a(\beta+1))^2}{16b(\beta+1)(b+\beta+1)(2b+\beta+1)} > 0.$$

(v) Restaurant's profit:

$$\pi_r^{DF*} - \pi_r^{DD*} = \frac{(8b-\beta-1)(a\beta+a-b)^2}{8b(b+\beta+1)(4b+\beta+1)^2}.$$

Hence, $\pi_r^{DF*} - \pi_r^{DD*} > 0$ if $b > \frac{1+\beta}{8}$; otherwise, $\pi_r^{DF*} - \pi_r^{DD*} \leq 0$. Additionally, we have

$$\pi_r^{DD*} - \pi_r^{FD*} = \frac{(2b^2+2(\beta+1)b+(\beta+1)^2)(a\beta+a-b)^2}{8b(\beta+1)(b+\beta+1)(2b+\beta+1)^2} > 0;$$
$$\pi_r^{DF*} - \pi_r^{FD*} = \frac{(16b^2+24(\beta+1)b+7(\beta+1)^2)(a\beta+a-b)^2}{4(\beta+1)(2b+\beta+1)^2(4b+\beta+1)^2} > 0.$$

(vi) Supply chain profit:

$$\pi_{sc}^{DF*} - \pi_{sc}^{FD*} = \frac{(8b^2+(\beta+1)b-(\beta+1)^2)(a\beta+a-b)^2}{4b(2b+\beta+1)^2(4b+\beta+1)^2}.$$

Hence, $\pi_{sc}^{DF*} - \pi_{sc}^{FD*} > 0$ when $8b^2 + (\beta+1)b - (\beta+1)^2 > 0$, that is, $b > \frac{\sqrt{33}-1}{16(1-\beta^2))}$; otherwise, $\pi_{sc}^{DF*} - \pi_{sc}^{FD*} \leq 0$. Additionally, we have

$$\pi_{sc}^{DF*} - \pi_{sc}^{DD*} = \frac{3(8b-\beta-1)(a\beta+a-b)^2}{16b(b+\beta+1)(4b+\beta+1)^2}.$$

Hence, $\pi_{sc}^{DF*} - \pi_{sc}^{DD*} > 0$ if $b > \frac{1+\beta}{8}$; otherwise, $\pi_{sc}^{DF*} - \pi_{sc}^{DD*} \leq 0$. Combining

$$\pi_{sc}^{FD*} - \pi_{sc}^{DD*} = \frac{(4b+\beta+1)(a\beta+a-b)^2}{16b(b+\beta+1)(2b+\beta+1)^2} > 0,$$

we have our results in Proposition B1 (vi). $\square$